INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2023

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

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SECTION A

QUESTION 1

(a) $m_{AB} = \frac{5+3}{1-7}$ $m_{CM} = \frac{3}{4}$

$= \frac{8}{-6}$ CM $\perp$ AB

$= \frac{-4}{3}$

(b) M (4; 1) $y - 1 = \frac{3}{4}(x - 4)$

$y - 1 = \frac{3}{4}x - 3$

$y = \frac{3}{4}x - 2$

$\therefore$ C (0, –2) (4)

(c) D (–6; 6)

(d) $CM^2 = (0-4)^2 + (-2-1)^2$ $AB^2 = (7-1)^2 + (-3-5)^2$

$= 16 + 9$ $= 36 + 64$

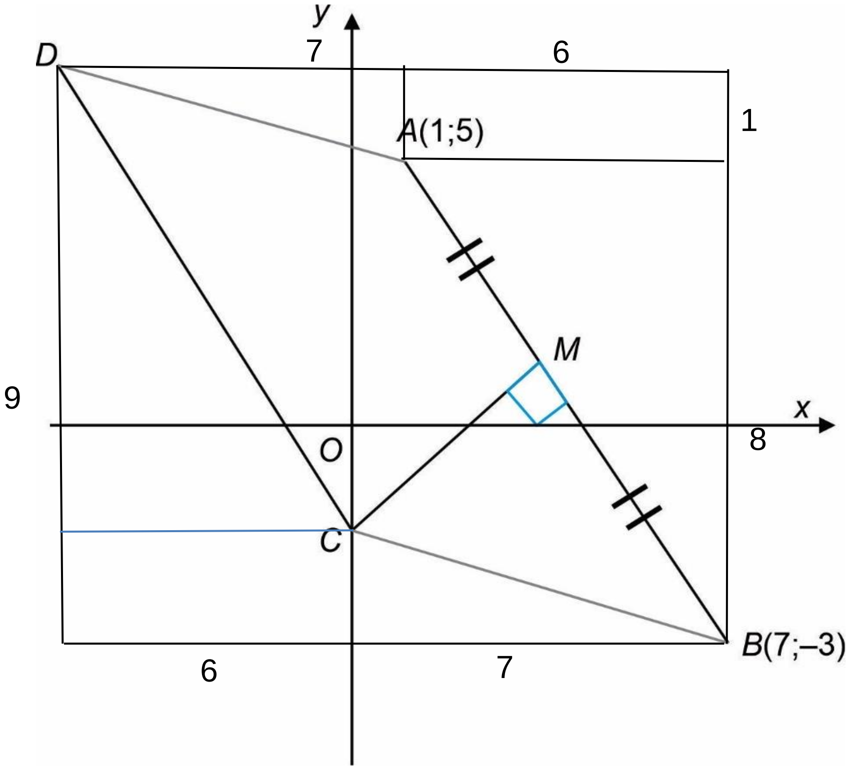
$= 25$ $= 100$

CM = 5 AB = 10

area of Parm = 10 × 5

= 50 units^2

OR ALTERNATE SOLUTION



area of Parm $= 9 \times 13 - 2\,(0{,}5 \times 7 \times 1) - 2(6 \times 1) - 2(0{,}5 \times 6 \times 8)$

$= 117 - 7 - 12 - 48$

$= 50$ units^2

(e) $m_{BC} = \dfrac{-2+3}{0-7}$

$= \dfrac{1}{-7}$

$\tan\theta_2 = \dfrac{-1}{7}$

$\theta_2 = 171{,}9^\circ$

$\therefore A\hat{B}C = 171{,}9^\circ - 126{,}9^\circ$

$= 45^\circ$

$\tan\theta_1 = \dfrac{-4}{3}$

$\theta_1 = 126{,}9^\circ$

OR ALTERNATE SOLUTION

CM = 5

AB = 10

BM = 5   M is midpoint

Thus   BM = CM

Triangle MCB isosceles

$\therefore A\hat{B}C = 45^\circ$ ($\angle$ opposite equal sides)

QUESTION 2

(a) $2x + x + 60^\circ = 180^\circ$ opp $\angle$ of cyclic quad

$3x = 120^\circ$

$x = 40^\circ$

$4(40^\circ) - 87^\circ + y = 180^\circ$ opp $\angle$ of cyclic quad

$y = 107^\circ$

(b) (1) $\hat{C} = 56^\circ$ $\angle$'s in same segm

(2) $\hat{O} = 112^\circ$ $\angle$ at centre = 2 $\angle$ at circumference

(3) $\hat{D}_1 + 56^\circ = 90^\circ$ $\angle$ in a semi-circle

$\hat{D}_1 = 34^\circ$

(4) $\hat{B}_1 = 34^\circ$ $\angle$'s in same segm

$\hat{B}_1 = \hat{B}_2$ given

$\therefore A\hat{E}D + 68^\circ = 180^\circ$ opp of cyclic quad

$A\hat{E}D = 112^\circ$

QUESTION 3

(a) (1) $\tan(180^\circ + \theta).\sin(90^\circ - \theta).\sin\theta + \cos^2(360^\circ - \theta)$

$= (\tan\theta)(\cos\theta)\sin\theta \quad + (+\cos\theta)^2$

$= \dfrac{\sin\theta}{\cos\theta} \times \cos\theta \times \sin\theta \quad + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

(2) $= \sin(x + 64^\circ).\cos(x + 379^\circ - 360^\circ) + \sin(x + 19^\circ)\cos(180^\circ + (64^\circ + x))$

$= \sin(x + 64^\circ)\cos(x + 19^\circ) + \sin(x + 19^\circ)(-\cos(64^\circ + x))$

$= \sin(x + 64^\circ)\cos(x + 19^\circ) - \sin(x + 19^\circ)\cos(x + 64^\circ)$

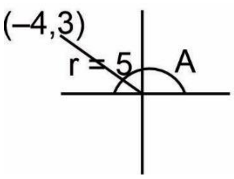
$= \sin[x + 64^\circ - (x + 19^\circ)]$

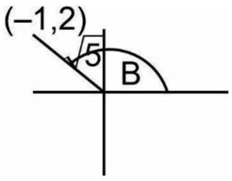
$= \sin 45^\circ$

$= \dfrac{\sqrt{2}}{2}$

(b) (1) II Quad

(2) $\dfrac{\cos A}{\sin B} = \dfrac{\frac{-4}{5}}{\frac{2}{\sqrt{5}}} \quad = \quad \dfrac{-4}{5} \times \dfrac{\sqrt{5}}{2}$





$= \dfrac{-2\sqrt{5}}{5}$

$5 = 1 + y^2$

$4 = y^2$

$y = 2$

(c) (1) LHS = $\dfrac{2\left(\cos^2 x - 1\right)}{\cos^2 x\left(1-\cos^2 x\right)}$

$= \dfrac{-2}{\cos^2 x}$

= RHS

(2) $\dfrac{-2}{\cos^2 x} = \dfrac{-4\sin x}{\cos x}$

$-2\cos x = 4\cos^2 x\sin x$

$0 = 4\cos^2 x\sin x + 2\cos x$

$0 = 2\cos x\,(2\cos x\,\sin x + 1)$

$2\cos x = 0$ OR $\sin 2x + 1 = 0$

$\cos x = 0$ $\sin 2x = -1$

$x = 90^\circ + k180^\circ$ $2x = -90^\circ + k360^\circ$

$(k \in z)$ $x = -45^\circ + k180^\circ$

OR

$2x = 270^\circ + k360^\circ$

$x = 135^\circ + k180^\circ$

$(k \in z)$

QUESTION 4

(a) $V = \pi r^2 h$

$75{,}4 = \pi r^2(6)$

$r^2 = 4{,}0$

$r = 2$ m

(b) ΔABC is equilateral

Area of $\Delta ABC = \frac{1}{2}(6)(6)\ \sin 60^{\circ}$

$= 15{,}5884\ m^2$

Area of $\odot = \pi(2)^2$

$= 12{,}5663\ m^2$

$\therefore$ Face Area $= 15{,}5884 - 12{,}5663$

$= 3{,}022$

$= 3$

(c) Area of Δ prism $= 3(6 \times 6) + 2(3)$

$= 108 + 6$

$= 114\ m^2$

Area of inner cylinder $= 2\pi rh$

$= 2\pi(2)(6)$

$= 75{,}3982\ m^2$

Total Area $= 189{,}4\ m^2$

(d) No of tins $= \frac{189{,}4}{9}$

$= 21{,}044$

$\therefore$ 21 tins of paint

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QUESTION 5

(a) (1) p = 12

q = 10

(2) (i) $\overline{x} = 132{,}6$

(ii) $\sigma = 19{,}0$

(3) It will be steeper, data is closer together

(b) (1) $\text{Mean} = \dfrac{p+6+2p}{3}$

$= \dfrac{3p+6}{3}$

$= p + 2$

(2) Variance would not change

SECTION B

QUESTION 6

(a) 360°

(b) 1

(c) 4 solution

(d) $g(x) = \cos 2x$ $\quad\quad$ $\cos^2 x - \sin^2 x = 0$

$\cos 2x = 0$

$\therefore\ g(x) = 0$ $\quad\quad$ So the x-intercepts

$\therefore\ x = 45^{\circ}$ or $x = 135^{\circ}$

(e) $g(x).f(x) \geq 0$

$x \in [30^{\circ}, 45^{\circ}]$ OR $[135^{\circ}, 180^{\circ}]$

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QUESTION 7

(a) Constr diam SOA and join A to R

$\hat{S}_1 + \hat{S}_2 = 90°$ rad $\perp$ tan

$\hat{R}_1 + \hat{R}_2 = 90°$ in a semi-circle

But $\hat{S}_2 = \hat{R}_1$ $\angle$'s in same segm

$\therefore \hat{S}_1 = \hat{R}_2$

OR $Q\hat{S}T = \hat{R}$

(b) (1) $\hat{X}_1 = \hat{Q}_1$ tan chd Th

$\hat{Q}_1 = \hat{Y}_1 + \hat{Y}_2$ ext $\angle$ of cyclic quad

$\therefore \hat{X}_1 = \hat{Y}_1 + \hat{Y}_2$ but these are alt $\angle$'s

(2) $\hat{Y}_2 = 30°$ ($\angle$'s in same segm): TXlly

but $\hat{Y}_1 + \hat{Y}_2 = 60°$ (alt $\angle$ on II lines)

$\therefore \hat{Y}_1 = 30°$ $\therefore$ QY bisects $X\hat{Y}Z$

(3) PQ II YZ (midpoint theorem)

$\hat{Y}_1 + \hat{Y}_2 = 60°$ (alt $\angle$ QP IIZY)

$\hat{Q}_3 = 180° - (60° + 30°)$ sum of $\angle$ in Δ

$\therefore \hat{Q}_3 = 90°$

$\therefore$ YZ is a diameter (subt 90º)

QUESTION 8

(a) (1) a = –17,1   b = 0,86

$y = -17{,}1 + 0{,}9x$

(2) r = 0,9908

r = 1

near-perfect correlation (positive)

(b) (1) 5 × 8 × 7 × 6 × 5 = 8 400

(2) If first digit is odd

3 × 7 × 6 × 5 × 4 = 2 520

Method if first digit is even

2 × 7 × 6 × 5 × 5 = 2 100

$\therefore$ total = 4 620

(c) $p(A \cap B) = p(A) \times p(B)$

$0{,}2 = x \times 0{,}3$

$x = \frac{2}{3}$

$p(A \text{ OR } B) = p(A) + p(B) - p(A \cap B)$

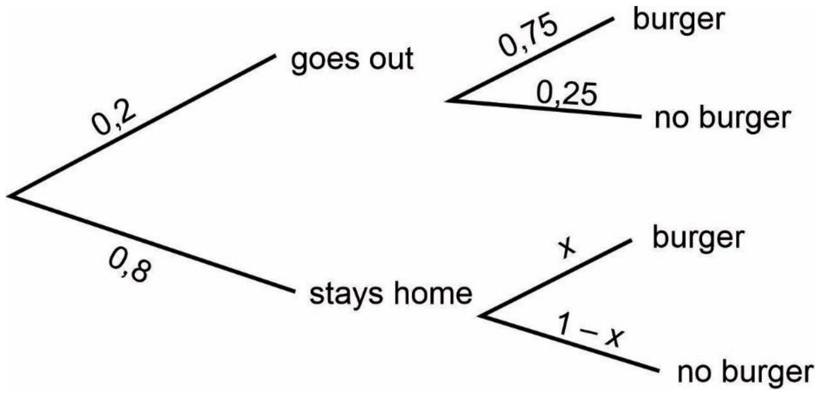
$= 0{,}67 + 0{,}3 - 0{,}2$

$= 0{,}77$

$\therefore\ y = 1 - 0{,}77$

$y = 0{,}23$

(d) (1)



(2) $0,2 \times 0,75 + 0,8x = 0,5$

$$0,15 + 0,8x = 0,5$$

$$0,8x = 0,35$$

$$x = \frac{3,5}{8}$$

$$= 0,4375$$

QUESTION 9

(a) (1) In ΔABC and ΔADO

$\hat{B} = 90^\circ$ ($\angle$ in a semi-circle) $\hat{D}_1 = 90^\circ$ (rad $\perp$ tan)

(1) $\hat{B} = \hat{D}_1$ proven

(2) $\hat{A} = \hat{A}$

(3) $\hat{C} = \hat{Q}$

$\therefore$ BAC ||| ΔDAO (A,A,A)

(2) $\hat{A} = 30^\circ$ ($\angle$ at centre = 2 $\angle$ at circum)

$\frac{DO}{AO} = \sin 30^\circ$

$\frac{5}{\frac{1}{2}} = AO$

$\therefore$ AO = 10

(3) Area Δ ADO = $\frac{1}{2}$ 5 × a = $\frac{5}{2}$a

Area Δ ABC = $\frac{1}{2}$(2a) 20 sin 30º

Area Quad DOCB = 10a – $\frac{5}{2}$a

= $\frac{15}{2}$a

= $\frac{10a}{\frac{15}{2}a}$

= 10 × $\frac{2}{15}$

= $\frac{4}{3}$

ALTERNATIVE

$$\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \perp \text{ Trapezium}} = \frac{\frac{1}{2} \times 10 \times 20a}{\frac{1}{2} \times 10a(10+5)} = \frac{4}{3}$$

(b) (1) In Δ CDT and ΔBTA

$$\frac{DT}{TA} = \frac{5}{16} = \frac{1}{2} \qquad \frac{CD}{AB} = \frac{9}{18} = \frac{1}{2} \qquad \frac{CT}{TB} = \frac{7}{14} = \frac{1}{2}$$

$\therefore$ ΔTCD|||ΔTBA (sides prop)

$\therefore C\hat{D}B = C\hat{A}B$

(2) Thus quad CBAD is cyclic

CB subt = $\angle$'s

$\therefore B\hat{D}A = B\hat{C}A$ ($\angle$'s in same segm)

QUESTION 10

(a) $AC^2 = 10^2 + 8^2$ Pythagoras

$= 100 + 64$

$= 164$

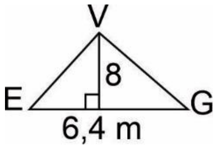
$\therefore\ AC = 12,8$

(b) $VM = 8$ $EM = 6,4$

$\tan\ V\hat{E}M = \dfrac{8}{6,4}$

$V\hat{E}M = 51,3^\circ$

(c) VE

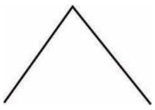


$VE^2 = 8^2 + 6,4^2$ Pythagoras

$VE^2 = 104,96$

$VE = 10,2$

(d) $E\hat{V}F$



$EF^2 = VE^2 + FV^2 - 2(VE)(VF)\cos E\hat{V}F$

$100 = 2(10,2)^2 - 2(10,2)^2\cos E\hat{V}F$

$208,08\cos E\hat{V}F = 108,08$

$\cos E\hat{V}F = 0,5194$

$E\hat{V}F = 58,7^\circ$

QUESTION 11

(a) $(-12,-15)$ $(24,12)$

$AC^2 = (24 + 12)^2 + (12 + 15)^2$

$= 36^2 + 27^2$

$= 2\ 025$

$AC = 45$

$5 + \text{diam} + 10 = 45$

$\text{diam} = 30$

$r = 15$ (5)

(b) (1) $$x^2 - kx + \left(\frac{-k}{2}\right)^2 = -c + \frac{k^2}{4} + \frac{(k+2)^2}{4}$$

$$\left(x - \frac{k}{2}\right)^2 + \left(y + \frac{k+2}{2}\right)^2 = \frac{-4c + k^2 + k^2 + 4k + 4}{4}$$

Centre $\left(\frac{k}{2}; \frac{-(k+2)}{2}\right)$

Sub in $2x + 5y + 14 = 0$

$$2\left(\frac{k}{2}\right) + 5\left(\frac{-k-2}{2}\right) + 14 = 0$$

$2k + 5(-k-2) + 28 = 0$

$-3k = -18$

$k = 6$

(2) $$r^2 = \frac{2k^2 + 4k + 4 - 4c}{4}$$

$\therefore r = \sqrt{25 - c}$ $$= \frac{72 + 24 + 4 - 4c}{4}$$

So $25 - c > 0$ $$= \frac{96 + 4 - 4c}{4}$$

$25 > c$ $= 24 + 1 - c$

$= 25 - c$

Total: 150 marks

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