INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
MEI 2023

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A

VRAAG 1

(a) $m_{AB} = \dfrac{5+3}{1-7}$ $\qquad$ $m_{CM} = \dfrac{3}{4}$

$= \dfrac{8}{-6}$ $\qquad$ CM $\perp$ AB

$= \dfrac{-4}{3}$

(b) M(4; 1) $\qquad$ $y - 1 = \dfrac{3}{4}(x - 4)$

$y - 1 = \dfrac{3}{4}x - 3$

$y = \dfrac{3}{4}x - 2$

$\therefore$ C(0, –2)

(c) D(–6; 6)

(d) $CM^2 = (0-4)^2 + (-2-1)^2$ $\qquad$ $AB^2 = (7-1)^2 + (-3-5)^2$

$= 16 + 9$ $\qquad$ $= 36 + 64$

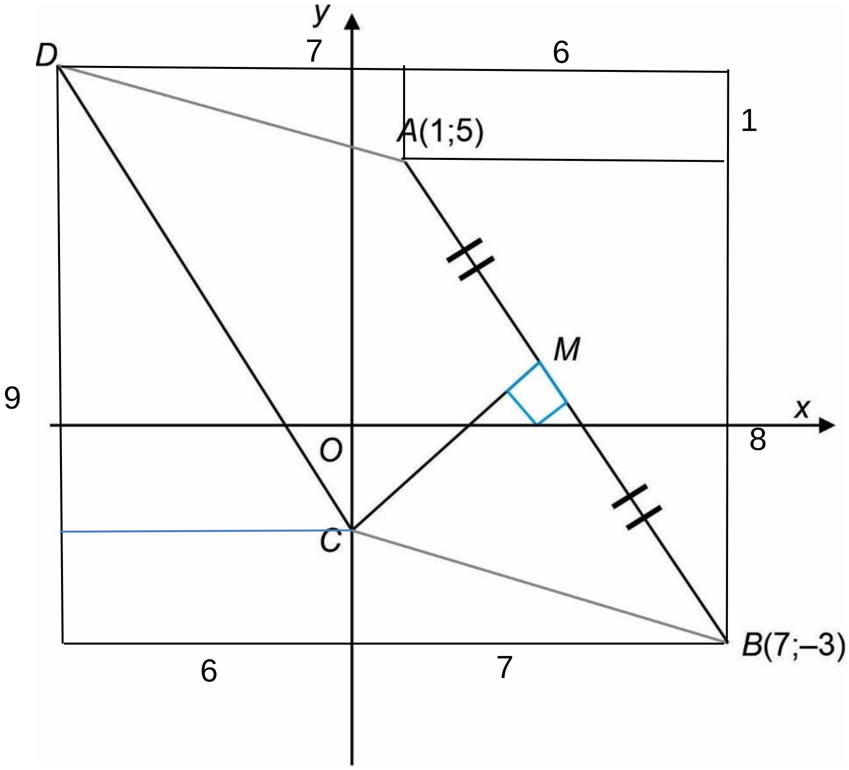
$= 25$ $\qquad$ $= 100$

CM = 5 $\qquad$ AB = 10

Oppervlakte van parm = 10 × 5

= 50 eenhede^2

OF ALTERNATIEWE OPLOSSING



Oppervlakte van parm $= 9 \times 13 - 2\,(0{,}5 \times 7 \times 1) - 2(6 \times 1) - 2(0{,}5 \times 6 \times 8)$

$= 117 - 7 - 12 - 48$

$= 50$ eenhede^2

(e) $m_{BC} = \dfrac{-2+3}{0-7}$

$= \dfrac{1}{-7}$

$\tan\theta_2 = \dfrac{-1}{7}$

$\theta_2 = 171{,}9^\circ$

$\therefore A\hat{B}C = 171{,}9^\circ - 126{,}9^\circ$

$= 45^\circ$

$\tan\theta_1 = \dfrac{-4}{3}$

$\theta_1 = 126{,}9^\circ$

OF ALTERNATIEWE OPLOSSING

CM = 5

AB = 10

BM = 5 M is middelpunt

Dus BM = CM

Driehoek MCB gelykbenig

$\therefore A\hat{B}C = 45^\circ$ ($\angle$ teenoor gelyke sye)

VRAAG 2

(a) $2x + x + 60^\circ = 180^\circ$ teenoorstaande $\angle$e van koordevierhoek

$3x = 120^\circ$

$x = 40^\circ$

$4(40^\circ) - 87^\circ + y = 180^\circ$ teenoorstaande $\angle$e van koordevierhoek

$y = 107^\circ$

(b) (1) $\hat{C} = 56^\circ$ $\angle$e in dieselfde segment

(2) $\hat{O} = 112^\circ$ $\angle$ by middelpunt = 2 maal $\angle$ by omtrek

(3) $\hat{D}_1 + 56^\circ = 90^\circ$ $\angle$ in halfsirkel

$\hat{D}_1 = 34^\circ$

(4) $\hat{B}_1 = 34^\circ$ $\angle$e in dieselfde segment

$\hat{B}_1 = \hat{B}_2$ gegee

$\therefore A\hat{E}D + 68^\circ = 180^\circ$ teenoorstaande $\angle$e van koordevierhoek

$A\hat{E}D = 112^\circ$

VRAAG 3

(a) (1) $\tan(180^\circ + \theta).\sin(90^\circ - \theta).\sin\theta + \cos^2(360^\circ - \theta)$

$= (\tan\theta)(\cos\theta)\sin\theta + (+\cos\theta)^2$

$= \dfrac{\sin\theta}{\cos\theta} \times \cos\theta \times \sin\theta + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

(2) $= \sin(x + 64^\circ).\cos(x + 379^\circ - 360^\circ) + \sin(x + 19^\circ)\cos(180^\circ + (64^\circ + x))$

$= \sin(x + 64^\circ)\cos(x + 19^\circ) + \sin(x + 19^\circ)(-\cos(64^\circ + x))$

$= \sin(x + 64^\circ)\cos(x + 19^\circ) - \sin(x + 19^\circ)\cos(x + 64^\circ)$

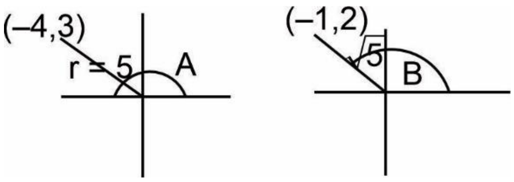
$= \sin[x + 64^\circ - (x + 19^\circ)]$

$= \sin 45^\circ$

$= \dfrac{\sqrt{2}}{2}$

(b) (1) Kwadrant II

(2) $\dfrac{\cos A}{\sin B} = \dfrac{\frac{-4}{5}}{\frac{2}{\sqrt{5}}} = \dfrac{-4}{5} \times \dfrac{\sqrt{5}}{2}$



$= \dfrac{-2\sqrt{5}}{5}$

$5 = 1 + y^2$

$4 = y^2$

$y = 2$

(c) (1) $\text{LK} = \dfrac{2\left(\cos^2 x - 1\right)}{\cos^2 x\left(1 - \cos^2 x\right)}$

$= \dfrac{-2}{\cos^2 x}$

$= \text{RK}$

(2) $\dfrac{-2}{\cos^2 x} = \dfrac{-4\sin x}{\cos x}$

$-2\cos x = 4\cos^2 x\sin x$

$0 = 4\cos^2 x\sin x + 2\cos x$

$0 = 2\cos x(2\cos x\sin x + 1)$

$2\cos x = 0$ OF $\sin 2x + 1 = 0$

$\cos x = 0$ | $\sin 2x = -1$

$x = 90^\circ + k180^\circ$ | $2x = -90^\circ + k360^\circ$

$(k \in z)$ | $x = -45^\circ + k180^\circ$

OF

$2x = 270^\circ + k360^\circ$

$x = 135^\circ + k180^\circ$

$(k \in z)$

VRAAG 4

(a) $V = \pi r^2 h$

$75{,}4 = \pi r^2(6)$

$r^2 = 4{,}0$

$r = 2$ m

(b) ΔABC is gelyksydig

Oppervlakte van $\Delta ABC = \frac{1}{2}(6)(6)\ \sin 60^\circ$

$= 15{,}5884\ m^2$

Oppervlakte van $\odot = \pi(2)^2$

$= 12{,}5663\ m^2$

$\therefore$ Vlakoppervlakte $= 15{,}5884 - 12{,}5663$

$= 3{,}022$

$= 3$

(c) Oppervlakte Δ prisma $= 3(6 \times 6) + 2(3)$

$= 108 + 6$

$= 114\ m^2$

Oppervlakte binnesilinder $= 2\pi rh$

$= 2\pi(2)(6)$

$= 75{,}3982\ m^2$

Totale oppervlakte $= 189{,}4\ m^2$

(d) Getal blikke $= \frac{189{,}4}{9}$

$= 21{,}044$

$\therefore$ 21 blikke verf

VRAAG 5

(a) (1) $p = \underline{\quad 12 \quad}$

$q = \underline{\quad 10 \quad}$

(2) (i) $\bar{x} = 132{,}6$

(ii) $\sigma = 19{,}0$

(3) Dit sal steiler wees, data is nader aan mekaar.

(b) (1) Gemiddelde $= \dfrac{p+6+2p}{3}$

$= \dfrac{3p+6}{3}$

$= p + 2$

(2) Variansie sal nie verander nie.

AFDELING B

VRAAG 6

(a) 360°

(b) 1

(c) 4 oplossings

(d) $g(x) = \cos 2x$ $\cos^2 x - \sin^2 x = 0$

$\cos 2x = 0$

$\therefore\ g(x) = 0$ Dus die x-afsnitte

$\therefore\ x = 45^\circ$ of $x = 135^\circ$

(e) $g(x).f(x) \geq 0$

$x \in [30^\circ, 45^\circ]$ OF $[135^\circ, 180^\circ]$

VRAAG 7

(a) Konstrueer middellyn SOA en verbind A met R

$\hat{S}_1 + \hat{S}_2 = 90°$ radius $\perp$ raaklyn

$\hat{R}_1 + \hat{R}_2 = 90°$ $\angle$ in halfsirkel

Maar $\hat{S}_2 = \hat{R}_1$ $\angle$e in dieselfde segment

$\therefore\ \hat{S}_1 = \hat{R}_2$

OF $Q\hat{S}T = \hat{R}$

(b) (1) $\hat{X}_1 = \hat{Q}_1$ raaklyn-koord-stelling

$\hat{Q}_1 = \hat{Y}_1 + \hat{Y}_2$ buite $\angle$ van koordevierhoek

$\therefore \hat{X}_1 = \hat{Y}_1 + \hat{Y}_2$ maar hulle is verw. $\angle$e

(2) $\hat{Y}_2 = 30°$ ($\angle$e in dieselfde segment): TX ll *y*

Maar $\hat{Y}_1 + \hat{Y}_2 = 60°$ (verw. $\angle$e op II lyne)

$\therefore \hat{Y}_1 = 30°$ $\therefore$ QY halveer $X\hat{Y}Z$

(3) PQ II YZ (middelpuntstelling)

$\hat{Y}_1 + \hat{Y}_2 = 60°$ (verw. $\angle$e QP II ZY)

$\hat{Q}_3 = 180° - (60° + 30°)$ som van $\angle$e in Δ

$\therefore\ \hat{Q}_3 = 90°$

$\therefore$ YZ is 'n middellyn (onderspan 90º)

VRAAG 8

(a) (1) a = –17,1 b = 0,86

y = –17,1 + 0,9x

(2) r = 0,9908

r = 1

byna perfekte korrelasie (positief)

(b) (1) 5 × 8 × 7 × 6 × 5 = 8 400

(2) Indien eerste syfer onewe is

3 × 7 × 6 × 5 × 4 = 2 520

Metode indien eerste syfer ewe is

2 × 7 × 6 × 5 × 5 = 2 100

$\therefore$ totaal = 4 620

(c) $p(A \cap B) = p(A) \times p(B)$

$0{,}2 = x \times 0{,}3$

$x = \frac{2}{3}$

p(A OF B) = p(A) +p(B) – $p(A \cap B)$

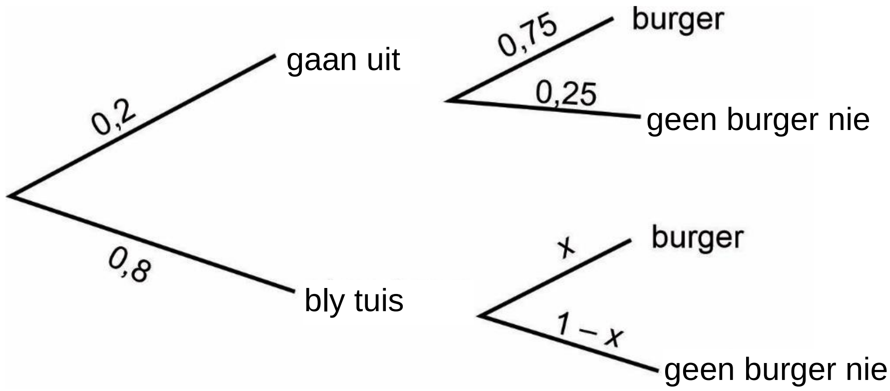
= 0,67 + 0,3 – 0,2

= 0,77

$\therefore$ y = 1 – 0,77

y = 0,23

(d) (1)



(2)
$$
\begin{aligned}
0{,}2 \times 0{,}75 + 0{,}8x &= 0{,}5 \\
0{,}15 + 0{,}8x &= 0{,}5 \\
0{,}8x &= 0{,}35 \\
x &= \frac{3{,}5}{8} \\
&= 0{,}4375
\end{aligned}
$$

VRAAG 9

(a) (1) In ΔABC en ΔADO

$\hat{B} = 90°$ ($\angle$ in halfsirkel) $\hat{D}_1 = 90°$ (radius $\perp$ raaklyn)

(1) $\hat{B} = \hat{D}_1$ bewys

(2) $\hat{A} = \hat{A}$

(3) $\hat{C} = \hat{Q}$

$\therefore$ BAC $|||$ ΔDAO (H,H,H)

(2) $\hat{A} = 30°$ ($\angle$ by middelpunt = 2 maal $\angle$ by omtrek)

$\frac{DO}{AO} = \sin 30°$

$\frac{5}{\frac{1}{2}}$ = AO

$\therefore$ AO = 10

(3) Oppervlakte ΔADO = $\frac{1}{2}$ 5 × a = $\frac{5}{2}$a

Oppervlakte ΔABC = $\frac{1}{2}$(2a) 20 sin 30º

Oppervlakte vierhoek DOCB = 10a – $\frac{5}{2}$a

= $\frac{15}{2}$a

= $\frac{10a}{\frac{15}{2}a}$

= 10 × $\frac{2}{15}$

= $\frac{4}{3}$

ALTERNATIEF

$$\frac{\text{Oppervlakte van } \Delta \text{ ABC}}{\text{Oppervlakte van} \perp \text{Trapesium}} = \frac{\frac{1}{2} \times 10 \times 20a}{\frac{1}{2} \times 10a(10+5)} = \frac{4}{3}$$

(b) (1) In ΔCDT en ΔBTA

$$\frac{DT}{TA}=\frac{5}{16}=\frac{1}{2} \qquad \frac{CD}{AB}=\frac{9}{18}=\frac{1}{2} \qquad \frac{CT}{TB}=\frac{7}{14}=\frac{1}{2}$$

$\therefore$ $\Delta TCD|||\Delta TBA$ (sye eweredig)

$\therefore C\hat{D}B = C\hat{A}B$

(2) Dus is CBAD koordevierhoek

CB onderspan gelyke $\angle$e

$\therefore$ $B\hat{D}A = B\hat{C}A$ ($\angle$e in dieselfde segment)

VRAAG 10

(a) $AC^2 = 10^2 + 8^2$ Pythagoras

$= 100 + 64$

$= 164$

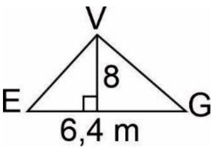
$\therefore\ AC = 12,8$

(b) $VM = 8 \qquad EM = 6,4$

$\tan\ V\hat{E}M = \dfrac{8}{6,4}$

$V\hat{E}M = 51,3^\circ$

(c) VE

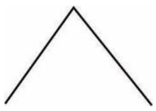


$VE^2 = 8^2 + 6,4^2$ Pythagoras

$VE^2 = 104,96$

$VE = 10,2$

(d) $E\hat{V}F$



$EF^2 = VE^2 + FV^2 - 2(VE)(VF)\cos E\hat{V}F$

$100 = 2(10,2)^2 - 2(10,2)^2 \cos E\hat{V}F$

$208,08 \cos E\hat{V}F = 108,08$

$\cos E\hat{V}F = 0,5194$

$E\hat{V}F = 58,7^\circ$

VRAAG 11

(a) (–12,–15) (24,12)

$AC^2 = (24 + 12)^2 + (12 + 15)^2$

$= 36^2 + 27^2$

$= 2\ 025$

$AC = 45$

5 + middellyn + 10 = 45

middellyn = 30

r = 15

(b) (1) $x^2 - kx + \left(\frac{-k}{2}\right)^2 = -c + \frac{k^2}{4} + \frac{(k+2)^2}{4}$

$\left(x - \frac{k}{2}\right)^2 + \left(y + \frac{k+2}{2}\right)^2 = \frac{-4c + k^2 + k^2 + 4k + 4}{4}$

Middelpunt $\left(\frac{k}{2}; \frac{-(k+2)}{2}\right)$

Vervang in $2x + 5y + 14 = 0$

$2\left(\frac{k}{2}\right) + 5\left(\frac{-k-2}{2}\right) + 14 = 0$

$2k + 5(-k - 2) + 28 = 0$

$-3k = -18$

$k = 6$

(2) $r^2 = \frac{2k^2 + 4k + 4 - 4c}{4}$

$\therefore r = \sqrt{25 - c}$ $= \frac{72 + 24 + 4 - 4c}{4}$

Dus $25 - c > 0$ $= \frac{96 + 4 - 4c}{4}$

$25 > c$ $= 24 + 1 - c$

$= 25 - c$

Totaal: 150 punte