



INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
NOVEMBER 2024

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A**VRAAG 1**

- (a) (i) 50
(ii) 5
(iii) 75% van 50 = 37,5
 $\pm 57\%$ (laat ± 2 toe)
- (b) (i) C
(ii) C

VRAAG 2

- (a) (i) $\frac{8!}{2! \times 2!} = 10\,080$
(ii) $\frac{2 \times 6!}{2!} \div 10\,080 = \frac{1}{14}$
- (b) (i) $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
(ii) $\left(\frac{1}{4} + \frac{1}{4}\right) - \frac{1}{16} = \frac{7}{16}$

VRAAG 3

- (a) Alle hoeke van gelyksydige $\Delta = 60^\circ$
- (b) Gearseerde oppervlakte = oppervlakte van groot Δ – oppervlakte van klein Δ
$$= \frac{1}{2}(80)(80)\sin 60^\circ - \frac{1}{2}(60)(60)\sin 60^\circ$$
$$= 1\,212,44 \text{ cm}^2$$

VRAAG 4

(a) (i) $D(4;0)$

(ii) $m_{DE} = \frac{-2-0}{-4-4} = \frac{1}{4}$

(iii) $y - 8 = \frac{1}{4}(x - 4)$
 $y = \frac{1}{4}x + 7$

(iv) $\tan \theta = \frac{1}{4}$
 $\theta = 14,04^\circ$
 $\hat{P}RB = 90^\circ - 14,04^\circ = 75,96^\circ$

(v) $Q(-4;6)$

(b) (i) Middelpunt van AB = (1;2)

(ii) $m_{AB} = \frac{3-1}{4+2} = \frac{1}{3}$

(iii) $\perp m = -3$
 $y - 2 = -3(x - 1)$
 $y - 2 = -3x + 3$
 $y = -3x + 5$

(iv) Middelloodlyn van AB gaan deur M.

$$y = -3x + 5 \text{ en } x = 0$$
$$\therefore y = -3(0) + 5 = 5$$
$$\therefore M(0;5)$$

(v) $r^2 = MB^2$ of $r^2 = MA^2$
 $= (4-0)^2 + (3-5)^2$ $= (-2-0)^2 + (1-5)^2$
 $= 20$

$$\therefore (x-0)^2 + (y-5)^2 = 20$$
$$x^2 + y^2 - 10y + 25 = 20$$
$$x^2 + y^2 - 10y + 5 = 0$$

VRAAG 5

$$(a) \quad \frac{(-2 \sin \theta)(-\sin 36^\circ)(\cos 36^\circ)}{\sin 72^\circ} = \frac{\sin \theta \sin 72^\circ}{\sin 72^\circ}$$

$$= \sin \theta$$

$$(b) \quad 2 \sin \theta \cos \theta - 4 \sin^2 \theta = 0$$

$$2 \sin \theta (\cos \theta - 2 \sin \theta) = 0$$

$$\sin \theta = 0^\circ \quad \text{of} \quad \tan \theta = \frac{1}{2}$$

$$\theta = 0^\circ + k \cdot 180^\circ \quad \theta = 26,57^\circ + k \cdot 180^\circ \quad k \in \mathbb{Z}$$

$$(c) \quad (i) \quad \frac{1 - 2 \sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{\sin^2 x + \cos^2 x - 2 \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$(ii) \quad \frac{1 - \sin 30^\circ}{\cos 30^\circ} = \frac{1 - \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

VRAAG 6

- (a) (i) $\frac{MN}{45} = \frac{108}{36}$
 $MN = 135$ Eweredigheidstelling PQ//LM
- (ii) In $\triangle NPQ$ en $\triangle NLM$:
 $\hat{N}$ is gemeenskaplik
 $\hat{NPQ} = \hat{NLM}$ ooreenkomstige $\angle$ e PQ // LM
 $\hat{PQN} = \hat{LMN}$ 3de $\angle$ van Δ of ooreenkomstige $\angle$ e PQ // LM
 $\therefore \triangle NPQ \sim \triangle NLM$ $\angle\angle\angle$
- $\frac{y}{x} = \frac{108}{72}$
 $= \frac{3}{2}$ of 1,5
- (b) (i) Raaklyn-koord-stelling
- (ii) $\hat{BAD} = 90^\circ - x$ raaklyn $\perp$ radius
 $\hat{E}_1 = 90^\circ - x$ buitehoek van koordevierhoek
- OF**
- $\hat{E}_3 = 90^\circ$ $\angle$ in halfsirkel
 $\hat{E}_2 = x$ $\angle$ e in dieselfde segment
 $\hat{E}_1 = 90^\circ - x$ $\angle$ e op reguitlyn
- (iii) $\hat{F}_1 = 90^\circ - x$ binne $\angle$ e van Δ
 $\hat{F}_1 = \hat{E}_1$
 $\therefore CFDE$ is koordevierhoek buite $\angle$ = teenoorstaande binne $\angle$

AFDELING B

VRAAG 7

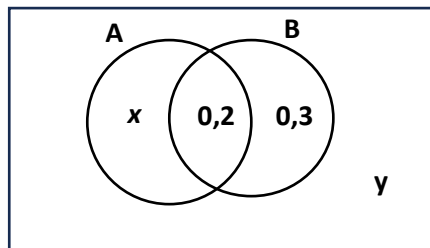
$$P(A \text{ en } B) = P(A) \times P(B)$$

$$0,2 = (x + 0,2)(0,5)$$

$$0,2 = 0,5x + 0,1$$

$$x = 0,2$$

$$y = 1 - 0,7 = 0,3$$



VRAAG 8

(a) $x = 30,78$

(b) $4,80$

(c) (i) $\frac{3 \times 5 + 9 \times 1}{12} = 2 \text{ }^{\circ}\text{C}$ per maand of tel 26 by totaal

(ii) Standaardafwyking sal afneem.
Variasiewydte sal afneem.

VRAAG 9

(a) $GA \parallel CD$
 $\hat{O}_1 = \hat{D}_1$

gegee

raaklyn-koord-stelling

$$\hat{O}_1 = \hat{C}$$

raaklyn-koord-stelling

$$\therefore \hat{D}_1 = \hat{C}$$

Maar hulle is ooreenkomstige $\angle$ e

$GC \parallel AD$

$AGCD$ is parm

albei pare teenoorstaande sye //

(b) $\hat{G}_1 = \hat{C}$

ooreenkomstige $\angle$ e $EA \parallel CO$

$$\hat{C} = \hat{D}_1$$

hierbo bewys

$$\hat{D}_1 = \hat{A}_2$$

verwisselende $\angle$ e $EA \parallel CO$

$$\hat{A}_2 = \hat{O}_2$$

raaklyn-koord-stelling

$\therefore GAOC$ is koordevierhoek

buite $\angle$ = teenoorstaande binne $\angle$

VRAAG 10(a) In $\triangle ABP$:

$$\frac{AR}{RB} = \frac{AS}{SP}$$

lyn $\parallel$ een sy van $\triangle$

$$\frac{AS}{SP} = \frac{4}{3}$$

(b) In $\triangle ABP$:

$$AP = PC = 7k$$

P as middelpunt gegee

$$\frac{AS}{SC} = \frac{4}{10} = \frac{2}{5}$$

(c) In $\triangle RSC$:

$$\frac{SP}{PC} = \frac{RT}{TC} = \frac{3}{7}$$

lyn $\parallel$ een sy van $\triangle$

$$\begin{aligned} \text{(d)} \quad \frac{\text{Oppervlakte } \triangle TPC}{\text{Oppervlakte } \triangle RSC} &= \frac{\frac{1}{2} TC \cdot PC \sin \hat{A}CR}{\frac{1}{2} RC \cdot SC \sin \hat{A}CR} \\ &= \frac{TC}{RC} \cdot \frac{PC}{SC} \\ &= \frac{7}{10} \times \frac{7}{10} \\ &= \frac{49}{100} \end{aligned}$$

VRAAG 11

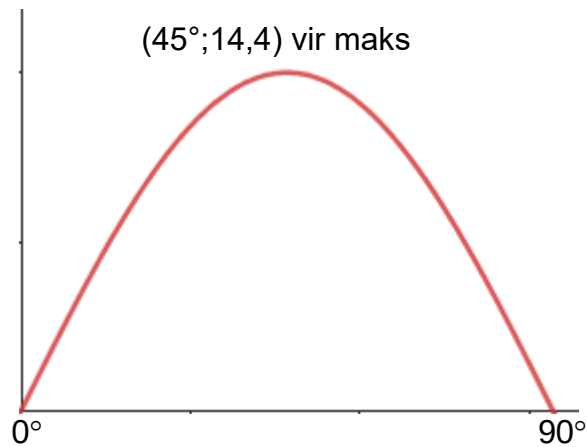
$$\text{(a)} \quad \frac{\sin E}{20} = \frac{\sin 30^\circ}{8}$$

$$\sin E = \frac{5}{4}$$

$$\text{maar } -1 < \sin E < 1$$

 $\therefore \triangle DEF$ bestaan nie $d \geq 10$, dus $d = 10$ is die minimum waarde sodat $\triangle DEF$ bestaan

$$\begin{aligned} \text{(b)} \quad \text{(i)} \quad y &= \frac{144}{10} \sin 2x \\ &= 14,4 \sin 2x \end{aligned}$$



$$\text{(ii)} \quad y \in [0; 14,4] \quad \text{of} \quad 0 \leq y \leq 14,4$$

$$\text{(iii)} \quad 14,4 \text{ m}$$

$$\begin{aligned} \text{(iv)} \quad 14,4 &= 14,4 \sin 2x \\ 1 &= \sin 2x \\ x &= 45^\circ \text{ (kan ook aflees van grafiek)} \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad 7,2 &= 14,4 \sin 2x \\ \sin 2x &= \frac{1}{2} \\ 2x &= 30^\circ \quad \text{of} \quad 2x = 150^\circ \\ x &= 15^\circ \quad \text{of} \quad x = 75^\circ \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad \frac{v^2}{10} \sin(2 \times 48^\circ) &= 23,37 \\ v^2 &= \frac{23,37 \times 10}{\sin 96^\circ} \\ v^2 &= 234,9872... \\ v &= 15,33 \text{ m/s} \end{aligned}$$

VRAAG 12

$$\begin{aligned} x^2 + 3x + \left(\frac{3}{2}\right)^2 + y^2 - 6y + (-3)^2 &= 9 + \frac{9}{4} + 9 \\ \left(x + \frac{3}{2}\right)^2 + (y - 3)^2 &= \frac{81}{4} \\ \text{radius van MP} &= \frac{9}{2} + 1 = \frac{11}{2} \\ \left(x + \frac{3}{2}\right)^2 + (y - 3)^2 &= \frac{121}{4} \end{aligned}$$

VRAAG 13

$$\text{Oppervlakte halfsirkel } ADC = \frac{1}{2}\pi\left(\frac{1}{2}AC\right)^2$$

$$\text{Oppervlakte halfsirkel } AB = \frac{1}{2}\pi\left(\frac{1}{2}AB\right)^2$$

$$\text{Oppervlakte halfsirkel } BC = \frac{1}{2}\pi\left(\frac{1}{2}BC\right)^2$$

Oppervlakte ADC – oppervlakte AB – oppervlakte BC

$$\begin{aligned} &= \frac{1}{2}\pi\left(\frac{1}{2}AC\right)^2 - \frac{1}{2}\pi\left(\frac{1}{2}AB\right)^2 - \frac{1}{2}\pi\left(\frac{1}{2}BC\right)^2 \\ &= \frac{1}{8}\pi(AC^2 - AB^2 - BC^2) \\ &= \frac{1}{8}\pi[(AB + BC)^2 - AB^2 - BC^2] \\ &= \frac{1}{8}\pi[AB^2 + 2AB \cdot BC + BC^2 - AB^2 - BC^2] \\ &= \frac{1}{8}\pi[2AB \cdot BC] \\ &= \frac{1}{4}\pi AB \cdot BC \quad \text{_____ (1)} \end{aligned}$$

Verbind AD en DC

$$\hat{ADC} = 90^\circ \quad \angle \text{ in halfsirkel}$$

In $\triangle ABD$ en $\triangle DBC$:

$$\hat{ABD} = \hat{DBC} = 90^\circ$$

$$\hat{A} = \hat{DC} \text{ want } \hat{D}_1 + \hat{D}_2 = 90^\circ \text{ en } \hat{A} + \hat{D}_1 = 90^\circ$$

$$\therefore \triangle ABD \sim \triangle DBC \quad \angle \angle \angle$$

$$\frac{AB}{DB} = \frac{BD}{BC} = \frac{AD}{DC}$$

$$BD^2 = AB \cdot BC$$

$$t^2 = AB \cdot BC \quad \text{_____ (2)}$$

Vervang (2) in (1):

$$\text{Oppervlakte} = \frac{1}{4}\pi t^2$$

Totaal: 150 punte