

INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
NOVEMBER 2022

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A**VRAAG 1**

$$\begin{aligned}
 \text{(a)} \quad RQ^2 &= (-5 - 10)^2 + (9 - 0)^2 \\
 &= 225 + 81 \\
 &= 306 \\
 RQ &= \sqrt{306} \text{ of } 17,5 \text{ of } 3\sqrt{34}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad M_{QR} &= \frac{9 - 0}{-5 - 10} \\
 &= \frac{9}{-15} \\
 &= \frac{-3}{5} \\
 y - 0 &= \frac{-3}{5}(x - 10) \text{ of } y - 9 = \frac{-3}{5}(x + 5) \\
 y &= \frac{-3}{5}x + 6
 \end{aligned}$$

$$\text{(c)} \quad S(0,6)$$

$$\begin{aligned}
 \text{(d)} \quad P(-8,0) \quad S(0,6) \quad M(-12,k) \\
 M_{PS} &= \frac{6 - 0}{0 + 8} \\
 &= \frac{6}{8} \\
 &= \frac{3}{4} \\
 \frac{3}{4} &= \frac{k - 6}{-12 - 0} \\
 -36 &= 4k - 24 \\
 -12 &= 4k \\
 k &= -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad &= \frac{\cancel{\frac{1}{2}} \times \cancel{18} \times 9}{\cancel{\frac{1}{2}} \times 10 \times 6} \\
 &= \frac{27}{10} \\
 &= 2,7
 \end{aligned}$$

VRAAG 2

(a) (1) $x = 60$
 $y = 74$

(2) Skeef na links

(b) (1) $n(s) = 40$ $n(g) = x$ $n(y) = 20$ $n(b) = 20 - x$

$$\begin{array}{ll} \frac{20-x}{40} = \frac{1}{8} & \text{of} \quad 40 \times \frac{1}{8} = 5 \\ 160 - 8x = 40 & \therefore 5 \text{ blou} \\ 120 = 8x & 40 - (20 + 5) = x \\ x = 15 & \end{array}$$

(2) $P(B) = \frac{15}{60}$
 $= \frac{1}{4}$

(3) $n(s) = 100$ $n(y) = 20 + 30$
 $p(y) = \frac{50}{100}$
 $= \frac{1}{2}$

(c) (1) $x = 13$ of hulle kan $x \geq 13$ skryf

$$x = 20$$

(2) $10 + 12 + 7 + 20 + 3 + 2$
 $= 54$

Dus benodig ons die 26^{ste} en 27^{ste} brokkie data
 Dus is mediaan 2

(3) $10 + 12 + 7 + x + 3 + 2 = 50$
 $34 + x = 50$
 $x = 16$
 $n = 50$

VRAAG 3(a) Konstruksie: verbind OD en OB

$$\hat{O}_1 = 2\hat{A} \quad (\angle \text{ by middelpunt} = 2\angle \text{ by sirkel})$$

$$\hat{O}_2 = 2\hat{C} \quad (\angle \text{ by middelpunt} = 2\angle \text{ by sirkel})$$

$$\hat{O}_1 + \hat{O}_2 = 360^\circ \quad (\angle \text{ e rondom 'n punt})$$

$$\therefore 2\hat{A} + 2\hat{C} = 360^\circ$$

$$\hat{A} + \hat{C} = 180^\circ$$

(b) (1) $\hat{BCD} = 180^\circ - (28^\circ + 36^\circ)$ teenoorstaande $\angle$ e van koordevierhoek
 $= 116^\circ$

(2) $\hat{B}_1 + \hat{B}_2 = 90^\circ$ radius $\perp$ raaklyn
 $\hat{O}_1 = 72^\circ$ $\angle$ by middelpunt = 2 $\angle$ by sirkel
 $\hat{E} = 180^\circ - (90^\circ + 72^\circ)$ $\angle$ e in Δ
 $\hat{E} = 18^\circ$

(3) $\hat{DBC} = 28^\circ$ ($\angle$ e in dieselfde segment)

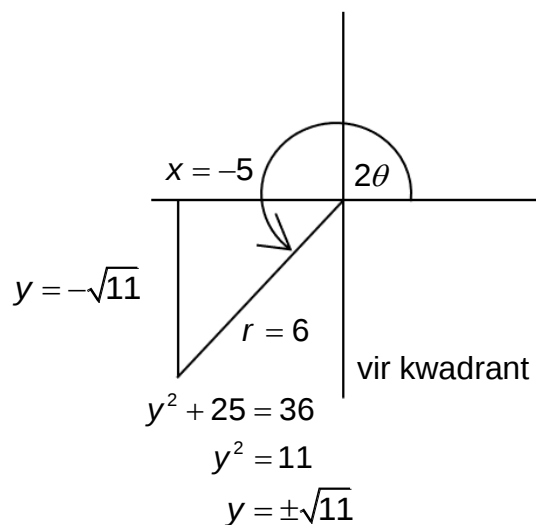
(4) $C_3 = 180^\circ - (18^\circ + 36^\circ)$ ($\angle$ e in Δ)
 $C_3 = 126^\circ$

VRAAG 4

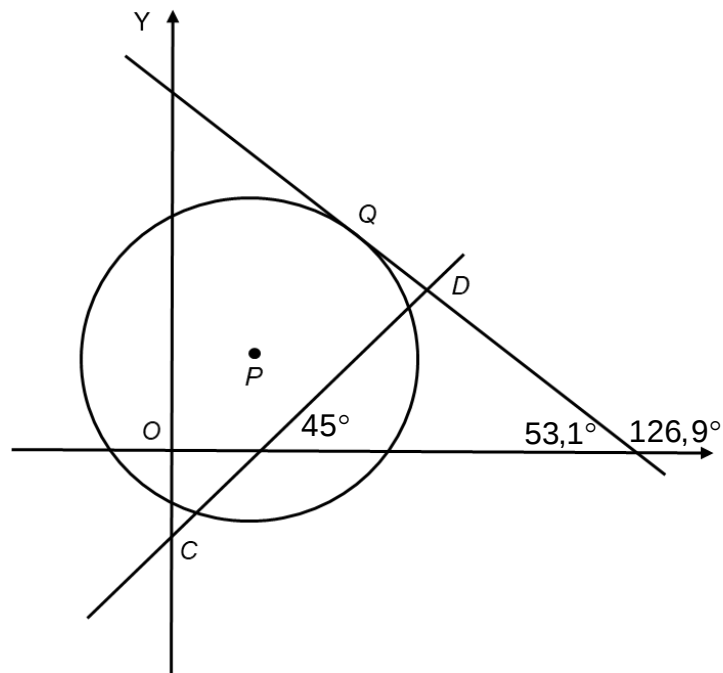
$$\begin{aligned}
 (a) \quad &= \frac{(-\sin x)(\tan x)^2(\cos x)}{(-\tan x)(\sin x)(\sin x)} \\
 &= \frac{\sin x}{\cos x} \times \frac{\cos x}{1} \quad \text{vir verandering van tan} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{LK} &= \cos^2 \theta \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right) \quad \text{verspreiding} \\
 &= \cos^2 \theta - \sin^2 \theta \\
 &= \cos 2\theta \\
 &= \text{RK}
 \end{aligned}$$

$$(c) \quad (1) \quad \sin 2\theta = \frac{-\sqrt{11}}{6}$$



$$\begin{aligned}
 (2) \quad \cos 2\theta &= 1 - 2\sin^2 \theta & \text{of} & \quad \cos 2\theta = 2\cos^2 \theta - 1 \\
 -\frac{5}{6} &= 1 - 2\sin^2 \theta & & \quad -\frac{5}{6} = 2\cos^2 \theta - 1 \\
 2\sin^2 \theta &= 1 + \frac{5}{6} & & \quad 2\cos^2 \theta = 1 - \frac{5}{6} \\
 2\sin^2 \theta &= \frac{11}{6} & & \quad 2\cos^2 \theta = \frac{1}{6} \\
 \sin^2 \theta &= \frac{11}{12} & & \quad 2(1 - \sin^2 \theta) = \frac{1}{6} \\
 & & & \quad 2 - 2\sin^2 \theta = \frac{1}{6} \\
 & & & \quad 2 - \frac{1}{6} = 2\sin^2 \theta \\
 & & & \quad \frac{11}{6} = 2\sin^2 \theta \\
 & & & \quad \sin^2 \theta = \frac{11}{12}
 \end{aligned}$$

VRAAG 5


(a) $x^2 - 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$
 $(x - 2)^2 + (y - 3)^2 = 25$
 middelpunt (2 ; 3) $r = 5$

(b) $LK = (7 - 2)^2 + (4 - 3)^2$
 $= 25 + 1$
 $= 26$

$LK > RK \therefore$ buite sirkel om toepaslike berekening te pas

(c) $t^2 + t^2 = 4t + 6t + 12$
 $2t^2 - 10t - 12 = 0$
 $t^2 - 5t - 6 = 0$
 $(t + 1)(t - 6) = 0$

$t = -1$ of $t = 6$
 NVT

(d) $M_{CD} = 1$
 $\tan \theta = 1$
 $\theta_1 = 45^\circ$

$\hat{QDC} = 53,1^\circ + 45^\circ$
 $= 98,1^\circ$ buite $\angle$ van driehoek

(2,3)	(6,6)
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$$M_{\text{rad}} = \frac{6 - 3}{6 - 2} = \frac{3}{4}$$

$$M_{\text{tan}} = -\frac{4}{3}$$

$$\tan \theta_2 = -\frac{4}{3}$$

$$\theta_2 = 126,9^\circ$$

OF

Teken parallelle lyn na x-as by D in

$$M_{CD} = 1$$

$$\tan \theta = 1$$

$$\theta_1 = 45^\circ$$

$$M_{rad} = \frac{6-3}{6-2} = \frac{3}{4}$$

$$M_{tan} = -\frac{4}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = 53,1^\circ$$

$$\begin{aligned}\hat{QDC} &= 53,1^\circ + 45^\circ \\ &= 98,1^\circ\end{aligned}$$

VRAAG 6

$$(a) \quad \frac{\text{oppervlakte van basis boks B}}{\text{oppervlakte van basis boks A}} = \frac{3\,375}{1\,500} = \frac{9}{4}$$

$$k^2 = \frac{9}{4}$$

$$\therefore k = \frac{3}{2}$$

$$(b) \quad A \times \frac{3}{2} = 99$$

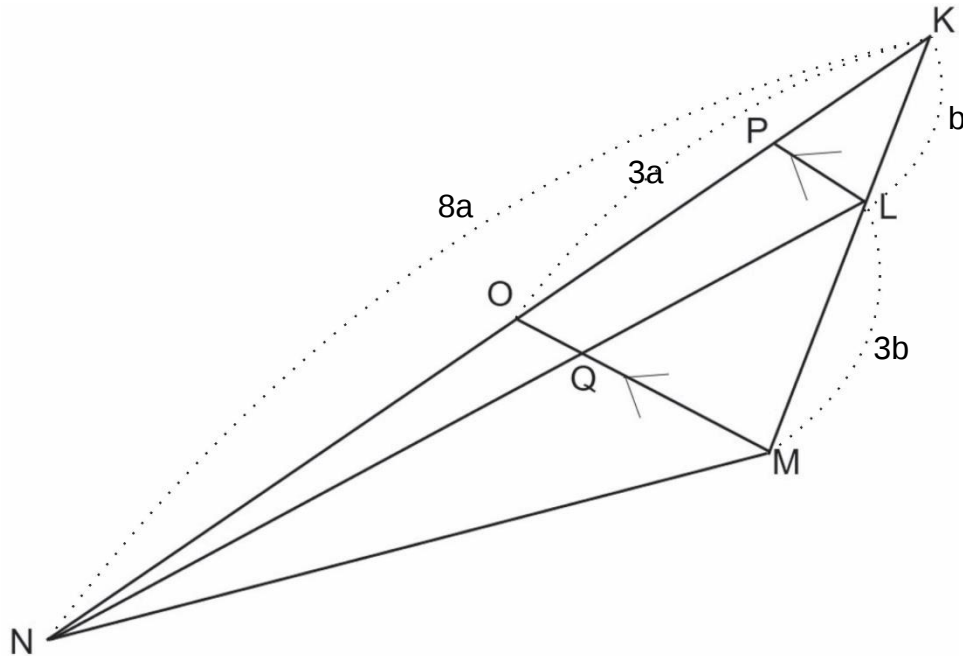
$$A = \frac{2}{3} \times 99$$

$$= -66 \text{ cm}$$

AFDELING B

VRAAG 7

- (a) "Die lyn wat parallel aan een sy van 'n driehoek getrek word, verdeel **die ander twee sye eweredig.**"
- (b)



$$\begin{aligned} \text{In } \triangle KOM \quad \frac{PO}{OK} &= \frac{LM}{KM} \quad (\text{lyn II een sy van } \triangle) \\ \frac{PO}{3a} &= \frac{3b}{4b} \\ \therefore PO &= \frac{9}{4}a \end{aligned}$$

$$\begin{aligned} \text{In } \triangle NPL \quad \frac{LQ}{QN} &= \frac{PO}{ON} \quad (\text{line II to one side of } \triangle) \\ \therefore \frac{LQ}{QN} &= \frac{9}{5} \\ &= \frac{9}{20} \end{aligned}$$

$$\begin{aligned} (2) \quad &= \frac{\frac{1}{2} \cdot 3a \cdot MO \cdot \sin \hat{KOM}}{\frac{1}{2} \cdot 5a \cdot MO \cdot \sin \hat{MON}} \end{aligned}$$

$$\begin{aligned} \text{Aangesien } \sin \hat{KOM} &= \sin \hat{MON} \\ &= \frac{3}{5} \end{aligned}$$

OF

Laat 'n loodregte hoogte van M na NK val. Noem dit h

$$\begin{aligned} & \frac{1}{2} \cdot h \cdot KO \\ = & \frac{\frac{1}{2} \cdot h \cdot NO}{2} \end{aligned}$$

$$= \frac{3}{5}$$

VRAAG 8

$$(a) \quad \cos x \cos 60^\circ - \sin x \sin 60^\circ + 2 \sin x = 0 = \frac{3}{5}$$

$$\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x + 2 \sin x = 0$$

$$\cos x = \sqrt{3} \sin x - 4 \sin x$$

$$\cos x = \sin x (\sqrt{3} - 4)$$

$$\tan x = \frac{1}{\sqrt{3} - 4}$$

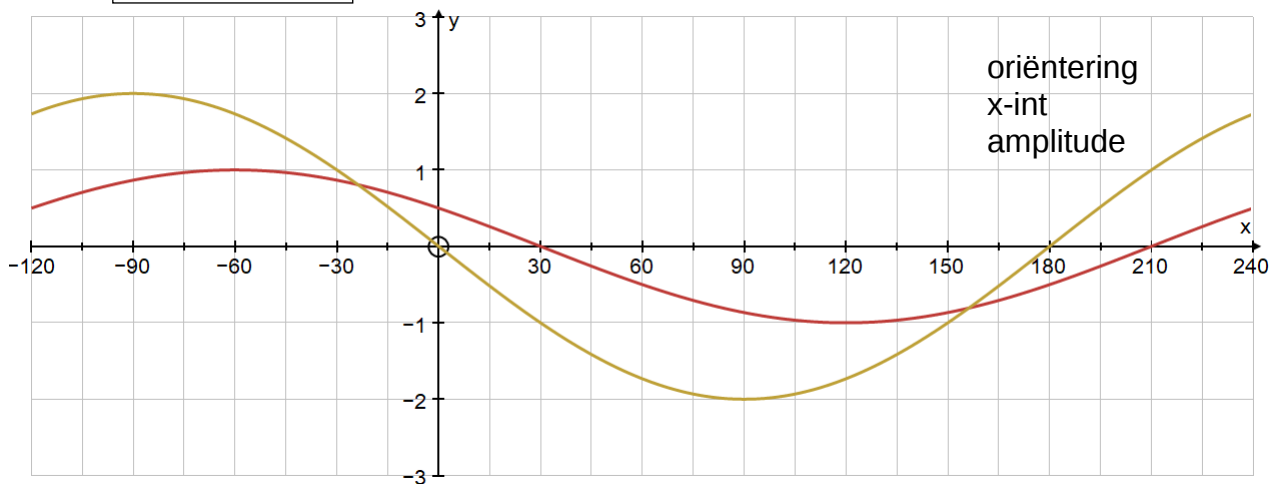
$$(b) \quad \tan x = \frac{1}{\sqrt{3} - 4}$$

$$x = -23,8^\circ + k180^\circ$$

$$x = -23,8^\circ \quad \text{of} \quad x = 156,2^\circ$$

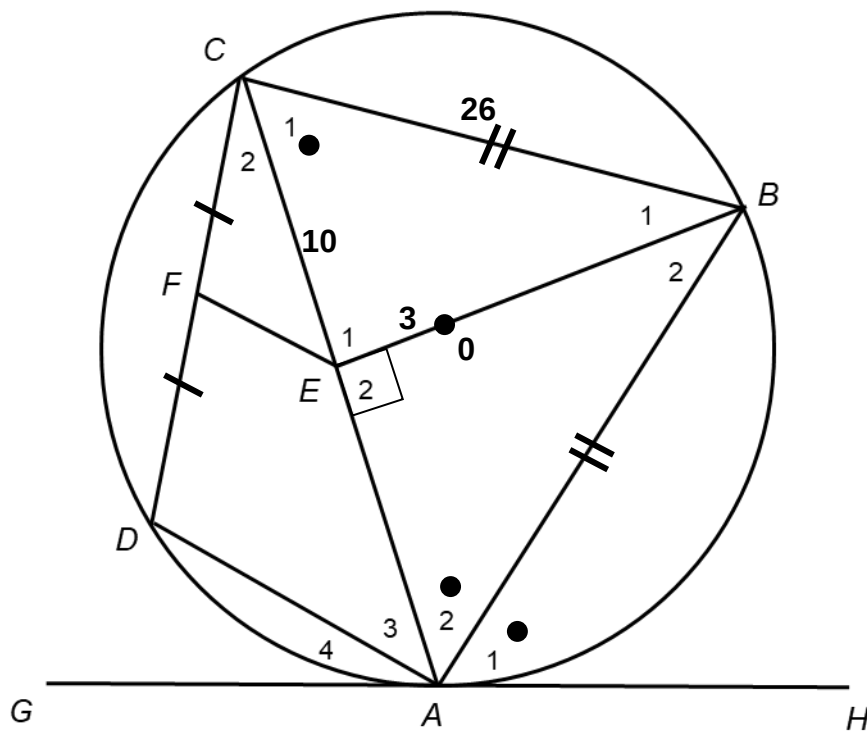
(c)

$$f(x) = -2 \sin x$$



$$(d) \quad x \in [-23,8^\circ, 156,2^\circ]$$

VRAAG 9



- (a) $\hat{A}_1 = \hat{C}_1$ raaklyn-koord-stelling
 $\hat{C}_1 = \hat{A}_2$ $\angle$ e teenoor gelyke sye
 $\therefore \hat{A}_1 = \hat{A}_2$
 $\therefore AB$ halveer $\hat{C}AH$
- (b) $CE = EA$ (lyn van middelpunt $\perp$ koord)
 $\therefore \frac{CF}{FD} = \frac{CE}{EA}$ (eweredige sye) of (middelpunt stelling)
 $\therefore FE \parallel DA$
- (c) $CE = EA = 10$ (hierbo bewys)
 laat $r = OB$
 $\therefore 26^2 = 10^2 + BE^2$ (geïmpliseerde gebruik van Pythagoras)
 $576 = BE^2$
 $BE = -24$ of $BE = 24$
 N/A $r = 21$
 $\therefore$ radius is 21 eenhede

OF

$$CE = EA = 10 \quad (\text{hierbo bewys})$$

Laat $OB = r$ Dus $EB = r + 3$

In driehoek CEB

$$10^2 + (r + 3)^2 = 26^2$$

$$100 + (r + 3)^2 = 576$$

$$(r + 3)^2 = 476$$

$$r + 3 = 21,8$$

$$r = 18,8$$

OF

$$CE = EA = 10 \quad (\text{hierbo bewys})$$

OC = r

In driehoek CEO

$$r^2 = 3^2 + 10^2$$

$$r^2 = 9 + 100$$

$$r^2 = 109$$

$$r = 10,4$$

VRAAG 10

$$43\,700\text{ cm}^3$$

$$\begin{aligned} V_{\text{sfeer}} &= \frac{1}{2} \cdot \frac{4}{3} \pi (23)^3 \\ &= 25\,482,51\text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V_{\text{silinder}} &= 43\,700 - 25\,482 \\ &= 18\,217,5 \end{aligned}$$

$$\pi r^2 h = 18\,217,5$$

$$h = \frac{18\,217,5}{\pi(23)^2}$$

$$h = 10,96\text{ cm}$$

OF

$$V = \frac{2}{3} \pi r^3 = \pi r^2 h$$

$$43700 = \frac{2}{3} \pi (23)^3 = \pi (23)^2 h$$

$$h = \frac{43700 - \frac{2}{3} \pi (23)^3}{\pi (23)^2}$$

$$h = 10,96$$

VRAAG 11

(a) (1) $p(A \cap B) = 0$
 $\therefore p(A) + p(B) = p(A \text{ of } B)$
 $0,4 + x = 0,9$
 $x = 0,5$

(2) $p(A) \cdot p(B) = p(A \cap B)$
 $p(A \text{ of } B) = p(A) + p(B) - p(A \text{ en } B)$
 $0,9 = 0,4 + x - 0,4x$ (vir $0,4x$) (vir vervanging)
 $0,5 = 0,6x$
 $x = \frac{5}{6}$

(b) (1) $= 13 \times 12 \times 11 \times 10 \times 9$ OF ${}^{13}P_5$
 $= 154\,440$

(2) $n(\text{geen leerders}) = 7 \times 6 \times 5 \times 4 \times 3$ OF 7P_5
 $= 2\,520$

$n(\text{minstens een leerder}) = 154\,440 - 2\,520$
 $= 151\,920$

(3) $n(\text{alle onderwysers}) = 10 \times 9 \times 3 \times 2 \times 1$
 $= 540$

(c) (1) $\bar{x} = 70,5$
 $\sigma = 14,8$ indien alles korrek is

(2) $70,5 - 14,8 < x < 70,5 + 14,8$
 $55,7 < x < 85,3$

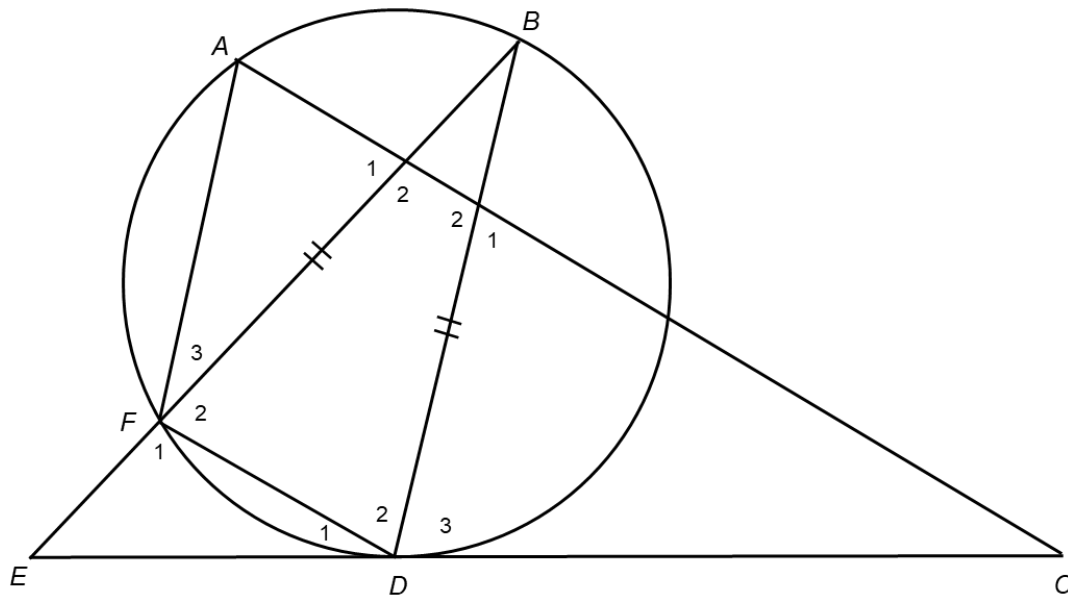
8 leerders lê binne een standaardafwyking

(3) $A = 16,4$ $B = 0,8$
 $y = 16,4 + 0,8x$
 $r = 0,932$

(4) $y = 16,4 + 0,8(55)$
 $= 60,4$

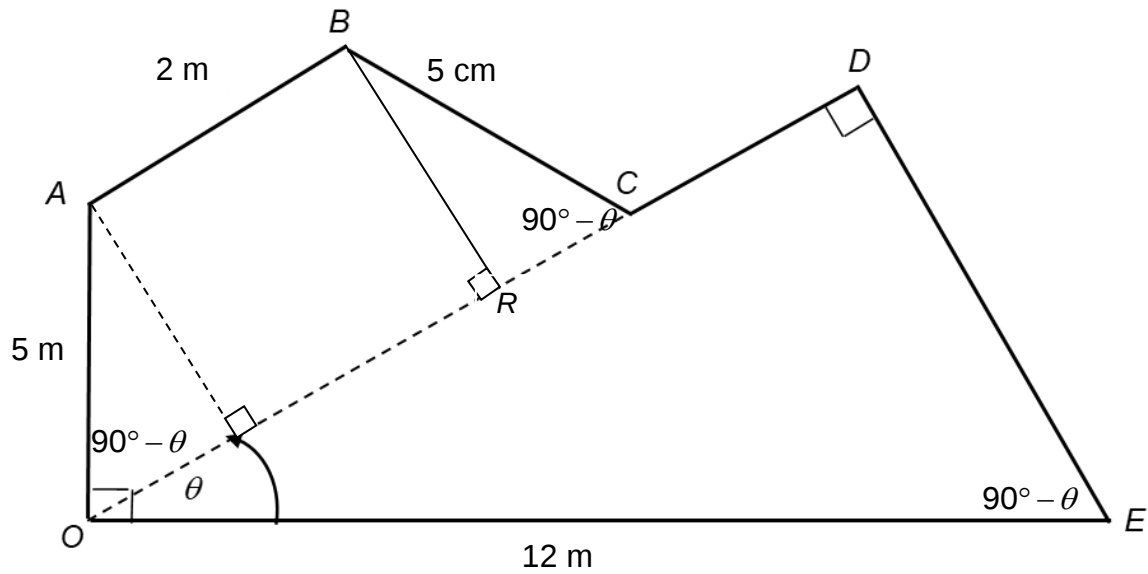
Aangesien die korrelasie baie sterk positief is, is dit 'n realistiese verwagting.

VRAAG 12



- (a) In $\triangle EFD$ en $\triangle EDB$
- | | | |
|---|-------------------------------|---------------------------------------|
| (1) | $\hat{E} = \hat{E}$ | gemeenskaplik |
| (2) | $\hat{D}_1 = \hat{B}$ | raaklyn-koord |
| (3) | $\hat{F}_1 = \hat{D}_1 + D_2$ | 3 ^{de} $\angle$ van Δ |
| $\therefore \triangle EDF \parallel \therefore \triangle EBD$ | | (HHH) |

(b) uit (a) $\frac{ED}{EB} = \frac{EF}{ED} = \frac{EF}{BD}$
 $ED^2 = EF \times EB$
 $= EF(EF + FB)$
 $= EF^2 + EF \times FB$
 maar $FB = BD$ gegee
 $\therefore ED^2 - EF^2 = EF \times BD$

VRAAG 13


$$\begin{aligned} \text{(a)} \quad \frac{RC}{5} &= \cos(90^\circ - \theta) \\ RC &= 5 \sin \theta \\ \therefore OC &= 2(5 \sin \theta) + 2 = 10 \sin \theta + 2 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{DE}{12} &= \sin \theta & S &= 5 + 2 + 5 + 12 + 12 \sin \theta + (12 \cos \theta - (10 \sin \theta + 2)) \\ DE &= 12 \sin \theta & S &= 24 + 12 \sin \theta + 12 \cos \theta - 10 \sin \theta - 2 \\ \frac{DO}{12} &= \cos \theta & S &= 22 + 2 \sin \theta + 12 \cos \theta \\ DO &= 12 \cos \theta \end{aligned}$$

Totaal: 150 punte