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TOTAL
MARKS

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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2023

MATHEMATICS: PAPER II

EXAMINATION NUMBER

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Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

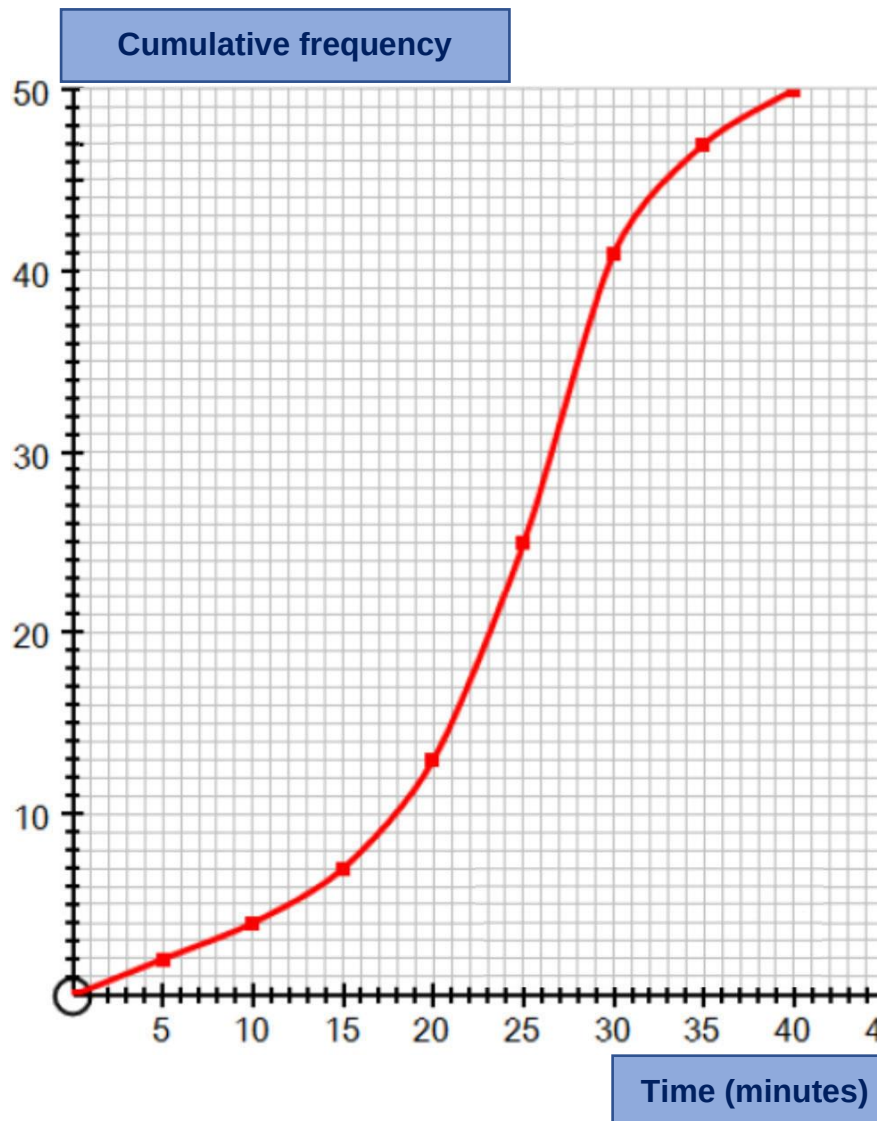
1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. **THREE** blank pages (pages 22 to 24) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
11	6	9	14	9	8	17	12	7	7	10	12	18	10	150

SECTION A**QUESTION 1**

The diagram below shows a cumulative frequency curve (ogive) which summarises the amount of time spent by 50 people answering their WhatsApps on a particular day:



Use the ogive to answer the following questions:

1.1 What was the median time spent answering WhatsApps?

Indicate with the letter A where you read this value from on the ogive.

(1)

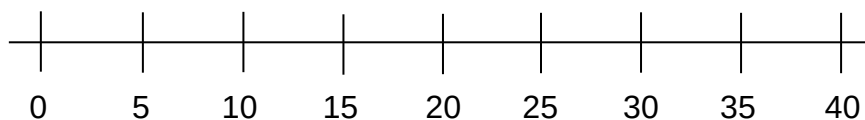
1.2 What was the interquartile range (IQR)?

Indicate with the letters B and C where you read the values from on the ogive. (3)

1.3 How many people spent between 15 and 20 minutes answering WhatsApps? (2)

1.4 If 80% of the people spent more than y minutes answering WhatsApps, what is the value of y ? (2)

1.5 Using the 5-number summary, draw a box-and-whisker plot for the data on the scale below: (3)



[11]

QUESTION 2

2.1 A group of ten businessmen visit Southern Africa. Of these, only 4 have previously been to Eswatini.

2.1.1 If one member of this group is chosen at random, what is the probability that the member of the group has never been to Eswatini before? (1)

2.1.2 If two members of this group are chosen at random, what is the probability that at least one of the two has visited Eswatini before? (2)

2.2 Events A and B are mutually exclusive. It is given that ...

$$3P(B) = P(A) \quad \text{and} \quad P(A \text{ or } B) = 0,63.$$

Calculate $P(B)$. (3)

[6]

QUESTION 3

A diameter AB of a circle with points A(-3 ; -2) and B(1 ; 4) is given.

3.1 Determine the equation of the circle. (4)

3.2 Determine the equation of the tangent at A. (5)

[9]

QUESTION 4

- 4.1 By making use of a sketch and without using a calculator, calculate the value of $\sin x$ if $15\cos x = -12$ and $\tan x > 0$. (3)

- 4.2 If $\sin 56^\circ = p$, express the following in terms of p without using a calculator:

4.2.1 $\sin(-236^\circ)$ (2)

4.2.2 $\cos 304^\circ$ (3)

4.3 Solve for x if $x \in [-180^\circ ; 90^\circ]$ and $\cos 2x = \sin(x - 30^\circ)$. (6)

[14]

QUESTION 5

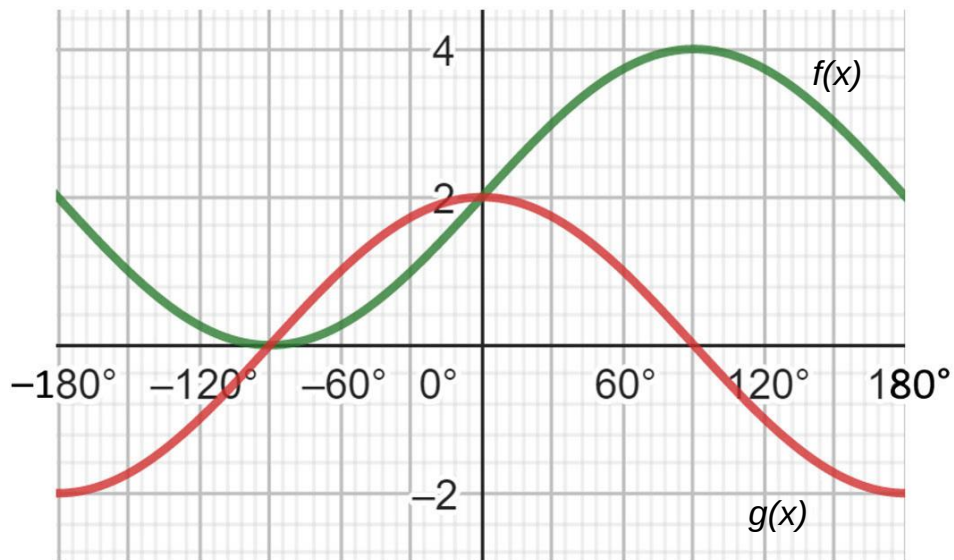
5.1 Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{1 - \cos x}{1 + \cos x}$. (5)

5.2 Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0^\circ \leq x \leq 360^\circ$. (4)

[9]

QUESTION 6

The graph in the figure represents the curves of $f(x) = p \sin x + q$ and $g(x) = r \cos x$ for $-180^\circ \leq x \leq 180^\circ$.



6.1 Determine the values of p , q , r and t in the given equations. (4)

6.2 Determine the period of $f(x)$. (1)

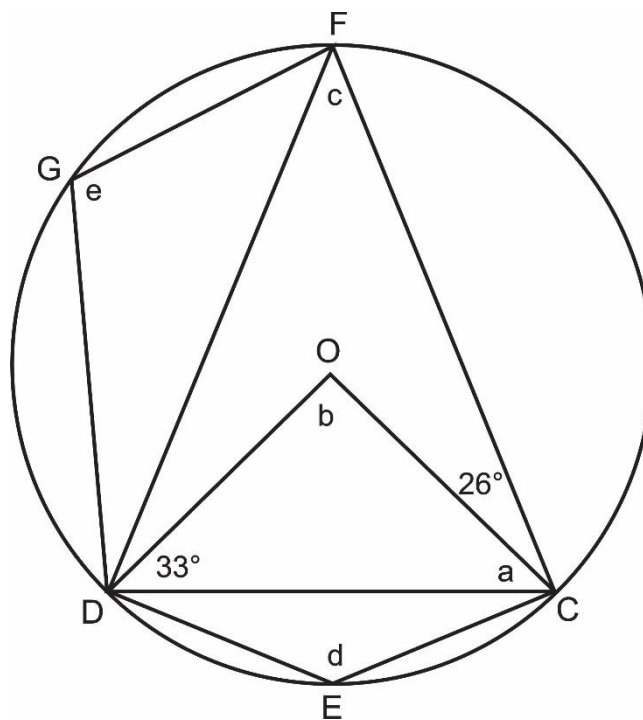
6.3 Determine the minimum value of $g(x)$. (1)

6.4 For what negative values of x is $f(x) > g(x)$ if $-180^\circ \leq x \leq 180^\circ$? (2)

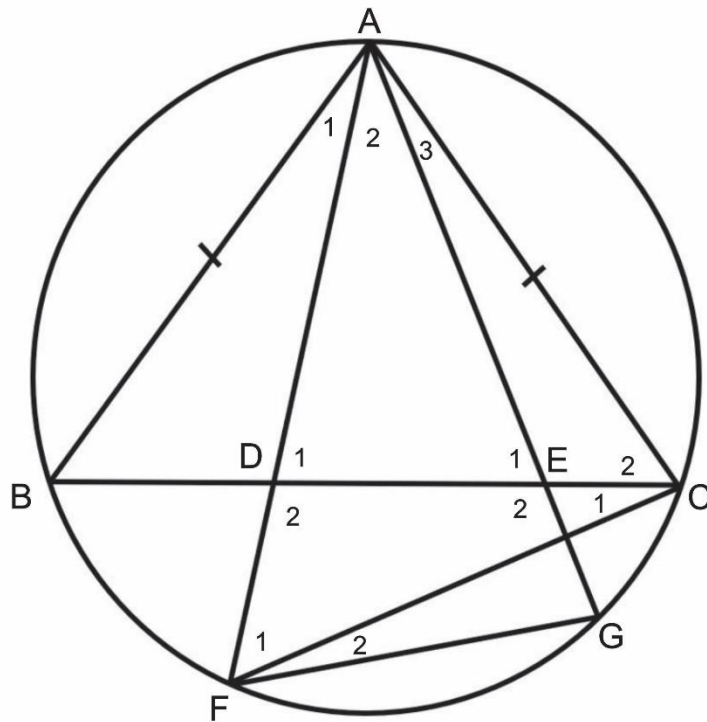
[8]

QUESTION 7

- 7.1 In the diagram, O is the centre of the circle. Find, giving reasons, the angles labelled **a** to **e**. (10)



- 7.2 In the diagram $AB = AC$ and D and E are points on BC . AF , AG and BC are straight lines. A , C , G , F and B are points on the circumference of the circle.

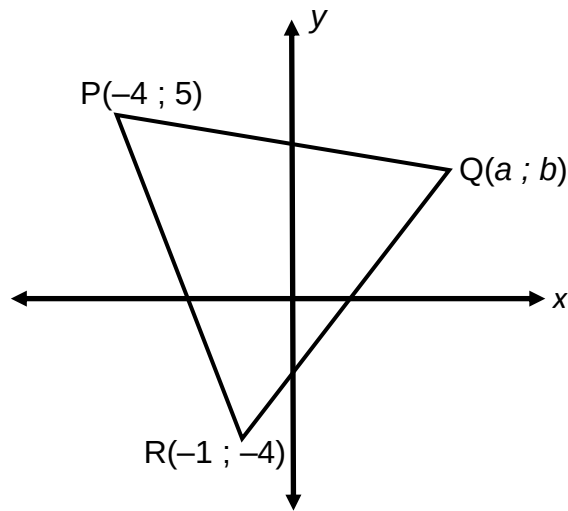


- 7.2.1 Prove that $\hat{G} = \hat{B} + \hat{A}_1$. (4)

- 7.2.2 Hence prove that $DFGE$ is a cyclic quadrilateral. (3)

[17]

74 marks

SECTION B**QUESTION 8**

In the figure $P(-4 ; 5)$, $Q(a ; b)$ and $R(-1 ; -4)$ are the vertices of a triangle and $M(1 ; 0)$ is the midpoint of QR .

8.1 Calculate the values of a and b . (2)

8.2 Determine the equation of the perpendicular bisector of QR . Give the equation in the form $ay + bx + c = 0$ where a , b and c are integers. (5)

- 8.3 If the perpendicular bisector of PQ has equation $7x - y + 8 = 0$, and cuts the perpendicular bisector of QR at the point T, calculate the co-ordinates of T. (5)

[12]

QUESTION 9

An IT company writes programs for apps. The time taken (in hours) to write the programs and the cost (in thousands of rands) are shown in the table below.

TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

- 9.1 Determine the equation of the least squares regression line. (2)
- 9.2 Use the equation of the least squares regression line to predict the cost, in rands, of an app that will take 16 hours to write. (2)
- 9.3 Calculate the correlation coefficient of the data. (1)
- 9.4 For each app that the company writes, there is a fixed cost that does not depend on the number of hours spent on writing the app. Calculate this cost (in rands). (2)

[7]

QUESTION 10

Consider the word FOOTBALL.

- 10.1 How many different arrangements formed by a random arrangement of the letters of the word FOOTBALL will start with the F and end with an L? (3)

- 10.2 What is the probability that an arrangement of the letters of FOOTBALL will have the letters F, B and A follow each other, in any order?

Give your answer in a fraction, in its simplest form. (4)

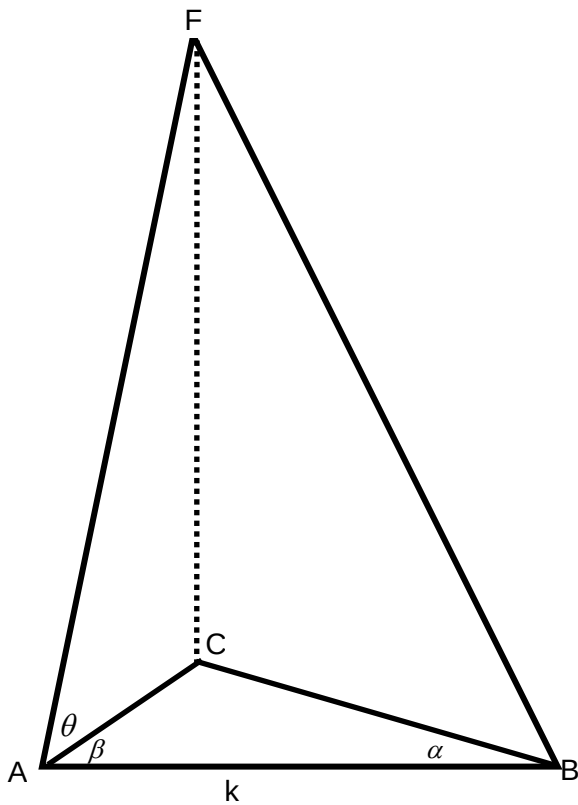
[7]

QUESTION 11

11.1 Simplify without using a calculator. Show all steps:

$$(\sin 15^\circ + \cos 15^\circ)^2 \quad (4)$$

11.2 In the picture of the Christus Kirche in Windhoek, points A, B and C are on flat ground.



C is a point immediately below the top of the cross on the gable, F.

The distance between A and B is k meters.

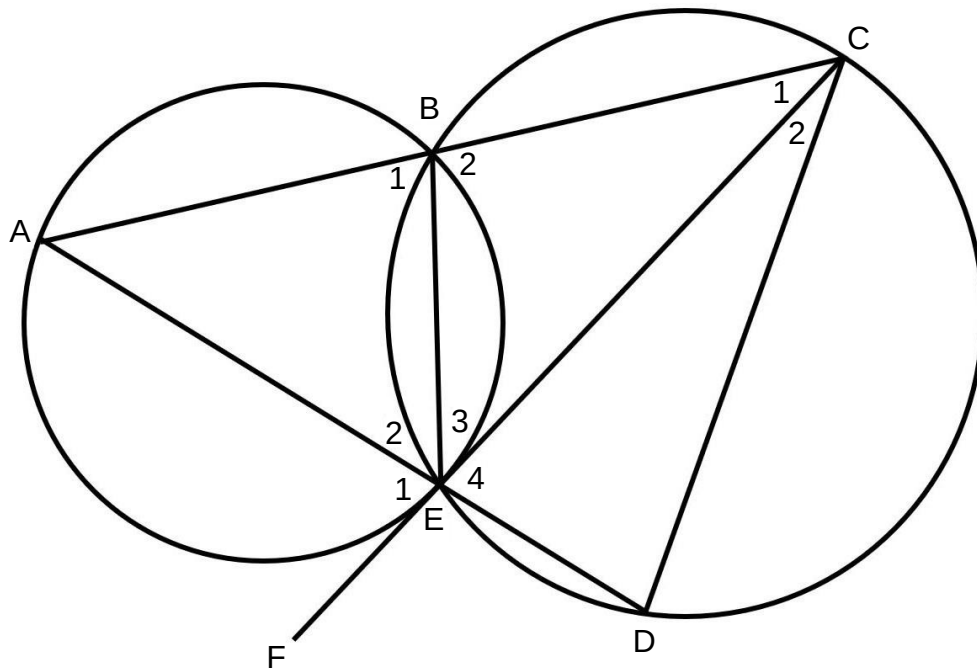
The angle of elevation of F from A is θ .

$$\hat{CAB} = \beta \quad \text{and} \quad \hat{CBA} = \alpha$$

Show that the height of the top of the cross is: $FC = \frac{k \sin \alpha \tan \theta}{\sin(\alpha + \beta)}$ (6)

QUESTION 12

In the diagram below CEF is a tangent to the smaller circle.



12.1 Prove that $CE = CD$.

(6)

12.2 Prove that $\triangle CBE \parallel \triangle CEA$.

(4)

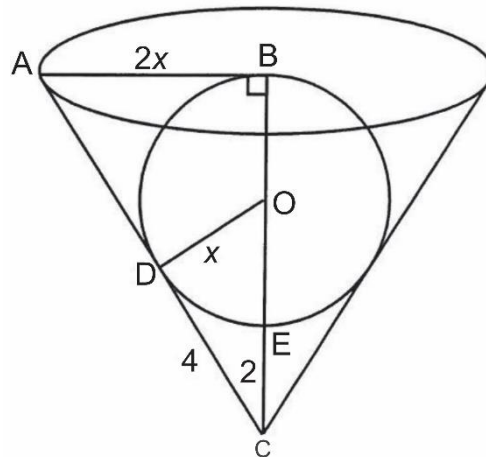
12.3 Hence, or otherwise, prove that $CE^2 = CA \cdot CB$.

(2)

[12]

QUESTION 13

- 13.1 A simple one-way valve can be created by using a conical funnel and a sphere that fits exactly inside it (i.e. the top of the sphere touches the horizontal plane that is the top of the upside-down cone), as illustrated in the diagram below (**not drawn to scale**):



A, B, C, D and O are in the same horizontal plane. O is the centre of the sphere and D is the point of contact between the sphere and the cone. $DO = x$ cm, $AB = 2x$ cm, $EC = 2$ cm and $DC = 4$ cm.

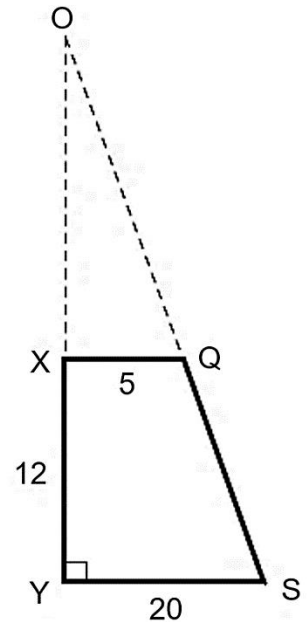
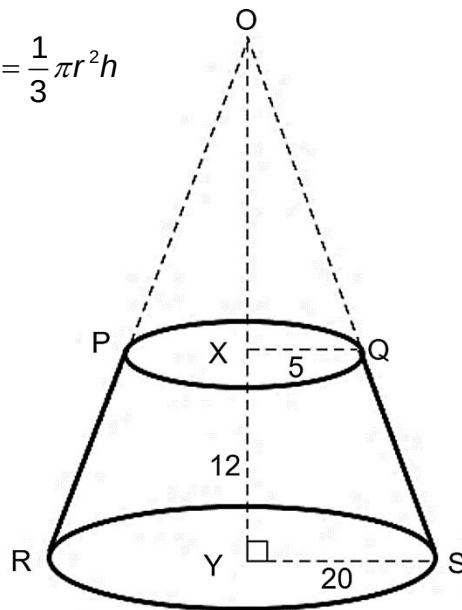
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 \text{ and Volume of cone} = \pi r^2 \frac{h}{3}.$$

- 13.1.1 Show that $x = 3$ cm, **giving all reasons:** (5)

- 13.1.2 Calculate the total volume of empty space remaining inside the funnel (above and below the sphere) when the sphere is in place, rounded to 1 decimal place: (4)

- 13.2 The figure shows a **frustum** which is obtained by removing the smaller cone OPQ with a base radius of 5 cm from the bigger cone ORS that has a base radius of 20 cm.

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$



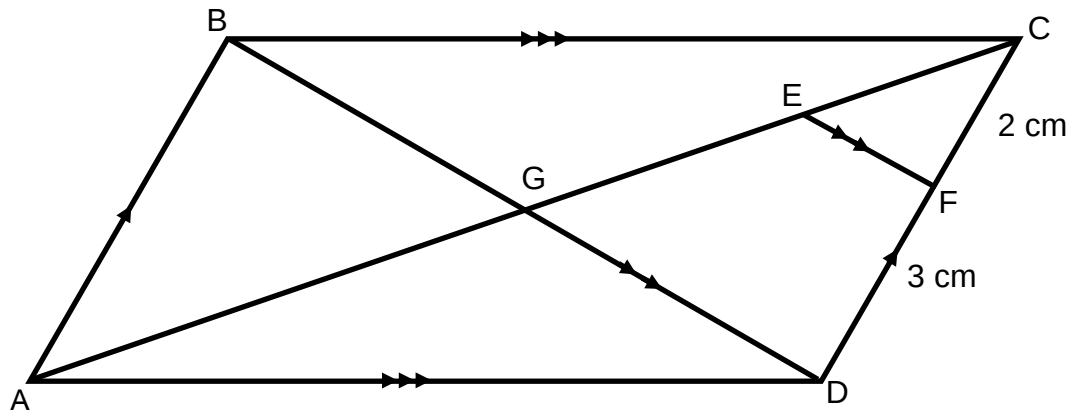
- 13.2.1 The height of the frustum, XY, is 12 cm. Calculate the height (OX) of the smaller cone, giving reasons. (5)

- 13.2.2 Find the volume of the frustum. Leave your answer in terms of π . (4)

QUESTION 14

In the diagram below ABCD is a parallelogram with the diagonals drawn.

$EF \parallel BD$, $CF = 2 \text{ cm}$ and $FD = 3 \text{ cm}$



14.1 Determine, giving reasons, the value of $\frac{CE}{CA}$. (4)

14.2 Determine, giving reasons, $\frac{\text{area } GEFD}{\text{area } ABCD}$. (6)

[10]

76 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

