INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2025

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

(a) (1) $\frac{1}{4}x^2 - 3x = 0$

$x\left(\frac{1}{4}x - 3\right) = 0$

$x = 0$ or $\frac{1}{4}x = 3$

$x = 12$

(2) $4.3^x - 3^{x+1} = 9$

$3^x(4-3) = 9$

$3^x = 3^2$

$x = 2$

(3) $2\log 3 + \log 2x - \log(3x-1) = 1$

$\log 9 + \log 2x - \log(3x-1) = 1$

$\log\left(\frac{9.2x}{3x-1}\right) = \log 10$

$18x = 10(3x-1)$

$18x - 30x = -10$

$-12x = -10$

$x = \frac{5}{6}$

(b) If $\log 3 = p$ and $\log 7 = q$, calculate, in terms of p and q, the value of $\log 4\frac{2}{7}$

$= \log\frac{30}{7}$

$= \log 30 - \log 7$

$= \log 10 + \log 3 - \log 7$

$= 1 + p - q$

(c) $\dfrac{2^{x+1}.18^{x-2}}{4^{x}.3^{2x-5}}$

$$= \frac{2^{x+1}.2^{x-2}.3^{2x-4}}{2^{2x}.3^{2x-5}}$$

$$= 2^{2x-1}.2^{2x}.3^{2x-4-2x+5}$$

$$= 2^{-1}.3$$

$$= \frac{3}{2}$$

QUESTION 2

(a) (1) $T_5 = 18$

$6 + 4d = 18$

$4d = 12$

$d = 3$

(2) $S_{20} = \frac{20}{2}\left[2(6) + 19(3)\right]$

$S_{20} = 690$

(b) (1) $T_2 = 12$ $\quad\quad T_5 = 1,5$

$ar = 12$ $\quad\quad ar^4 = 1,5$

$a = \frac{12}{r}$ $--> \quad \frac{12r^4}{r} = 1,5$

$r^3 = \frac{1,5}{12} = \frac{1}{8}$

$<-- r = \frac{1}{2}$

$a = \frac{12}{\left(\frac{1}{2}\right)}$

$a = 24$

$a = 24\ ;\ r = \frac{1}{2}$

(2) $S_\infty = \frac{24}{1 - \frac{1}{2}}$

$S_\infty = 48$

QUESTION 3

(a) $$f'(x)=\lim_{h\to 0}\frac{\frac{1}{2}\left(x+h\right)^2+x+h-\left(\frac{1}{2}x^2+x\right)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2+x+h-\frac{1}{2}x^2-x}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{h\left(x+\frac{1}{2}h+1\right)}{h}$$

$$f'(x)=x+\frac{0}{2}+1$$

$$f'(x)=x+1$$

(b) $$f(x)=2x^5-4x^{-3}+x^{\frac{2}{5}}$$

$$f'(x)=10x^4+12x^{-4}+\frac{2}{5}x^{-\frac{3}{5}}$$

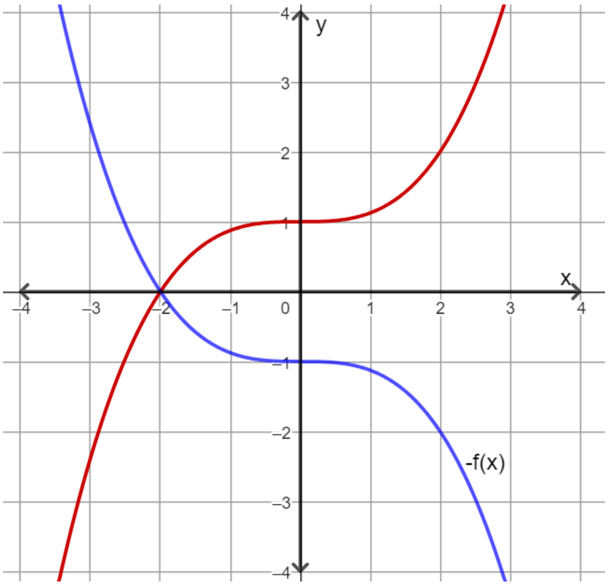
(c) $f(3)=10\therefore(3;10)$ *lies on* f.

$f^{-1}(4)=2\therefore(2;4)$ *lies on* f.

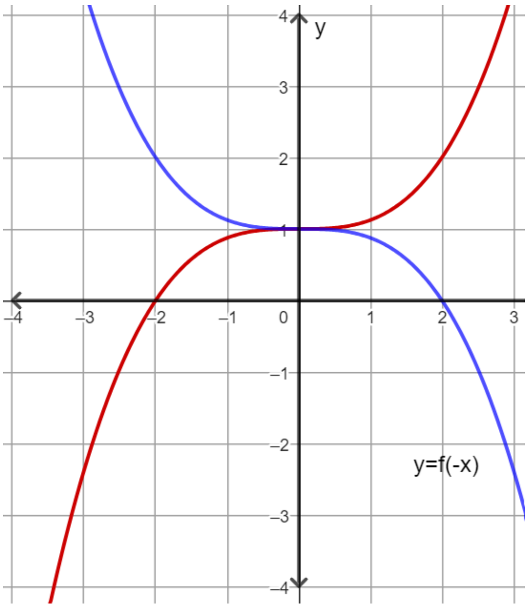
$$gradient=\frac{10-4}{3-2}=6$$

QUESTION 4

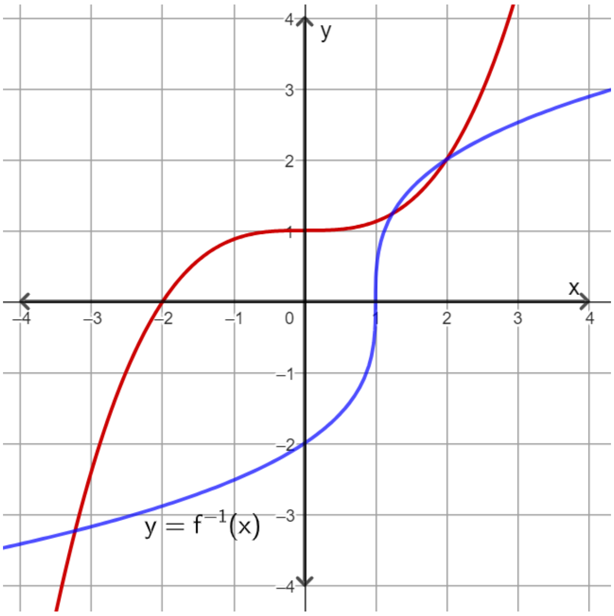
(a) $y = -f(x)$



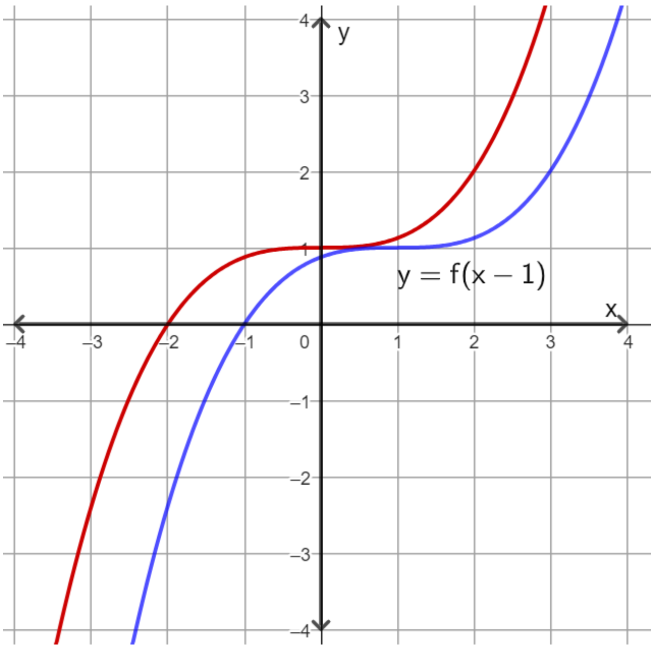
(b) $y = f(-x)$



(c) $y = f^{-1}(x)$



(d) $y = f(x-1)$



QUESTION 5

(a) $A = 350000(1+0{,}069)^5$

$A = 488\ 603{,}4964\ \ (accurate)\ or\ A = 488\ 603{,}50$

(b) $0{,}08 = \left(1+\frac{r}{12}\right)^{12} - 1$

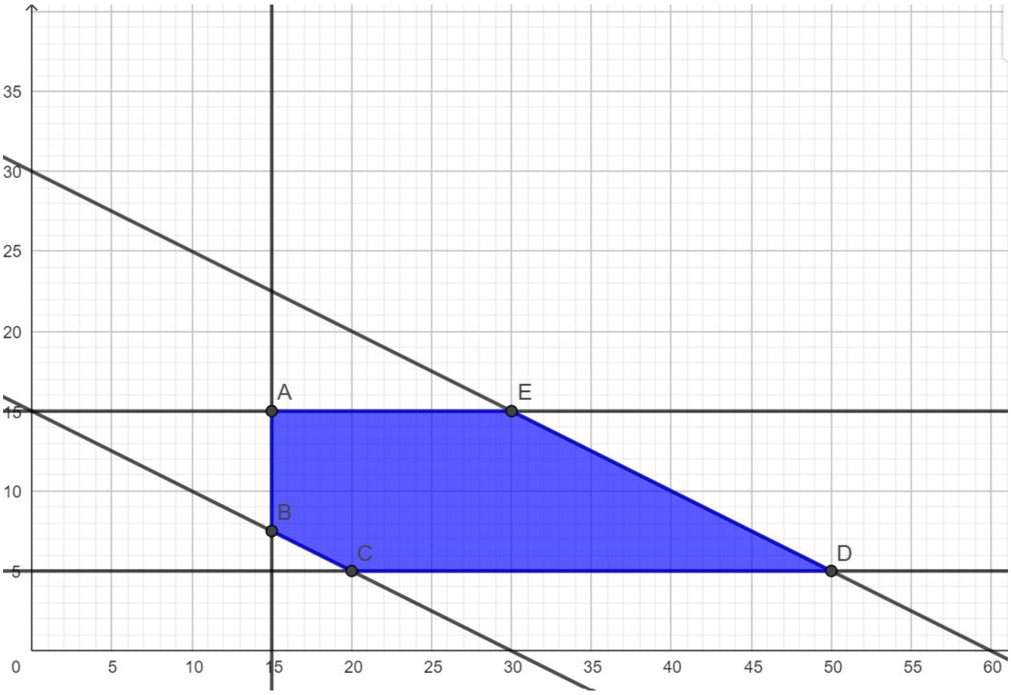
$r = 0{,}0772....$

(c) $488603{,}50 = x\frac{\left[\left(1+\frac{0{,}0772..}{12}\right)^{61} - 1\right]}{\frac{0{,}0772..}{12}}$

$x = 6566{,}02\ (accurate)\ or\ 6566{,}16(4dp)$

QUESTION 6

(a)



Each line feasible region

(b) (1) $P = 15x + 60y$

$y = -\frac{15}{60}x + \frac{P}{60}$

$\therefore m = -\frac{1}{4} \quad -> \ gradient$

$\therefore x = 30 \ and \ y = 15$ (using search line)

(2) $P_{\max} = 15(30) + 60(15)$

$P_{\max} = 1350$

QUESTION 7

(a) $N = 10 - 3 = 7 \quad \textit{years}$

$\therefore 7 \times 12 = 84 \quad \textit{payments left}$

$$289\ 985{,}16 = x\left[\frac{1-\left(1+\frac{14\%}{12}\right)^{-84}}{\frac{14\%}{12}}\right]$$

$x = 5434{,}33 \quad \textit{monthly}$

(b) $n = 3 \times 12 = 36$

$$289\ 985{,}16 = P\left(1+\frac{14\%}{12}\right)^{36} - 5\ 434{,}33\left[\frac{\left(1+\frac{14\%}{12}\right)^{36}-1}{\frac{14\%}{12}}\right]$$

$P = 350\ 000 \quad \textit{Original loan}$

QUESTION 8

(a) $26 \quad ; \qquad p; \qquad p^2; \qquad 34$

$$p + 26; \qquad p^2 - p; \qquad 34 - p^2$$

$$p^2 - p - p - 26 = 34 - p^2 - p^2 + p$$
$$\therefore p^2 - 2p - 26 = -2p^2 + p + 34$$
$$\therefore 3p^2 - 3p - 60 = 0$$
$$\therefore p = 5 \quad or \quad p = -4$$

(b) $\sum_{k=1}^{m} \frac{1}{25}(5)^{k-1} < 6$

$$\frac{1}{25} + \frac{1}{5} + 1 + 5 + ...$$

Geometric $\quad r = 5$ *and* $a = \frac{1}{25}$

$$S_n = \frac{\frac{1}{25}\left(5^m - 1\right)}{5 - 1}$$
$$\frac{1}{100}\left(5^m - 1\right) < 6$$
$$\left(5^m - 1\right) < 600$$
$$5^m < 601$$
$$m < \log_5 601$$
$$m < 3{,}97$$
$$\therefore m = 3$$

QUESTION 9

(a) $x > 2$

(b) $sub(2;1)$

$$1 = \sqrt{a(2)} - 1$$
$$2 = \sqrt{a(2)}$$
$$4 = 2a$$
$$\therefore a = 2$$

$$c = 3$$
$$g(x) = b.2^x + 3 \text{ now } sub(2;1)$$
$$1 = b.2^2 + 3$$
$$b = -\frac{1}{2}$$

(c) $(x) = -\frac{1}{2}.2^x + 3$

$$g^{-1}(x): \text{swap } x \text{ and } y$$
$$x = -\frac{1}{2}.2^y + 3$$
$$x - 3 = -\frac{1}{2}.2^y$$
$$6 - 2x = 2^y$$
$$\therefore y = \log_2(6 - 2x)$$

QUESTION 10

(a) Slopes equal at same x

$$\frac{dy}{dx} = 4x - 3$$
$$\frac{dy}{dx} = -2x + 3$$
$$4x - 3 = -2x + 3$$
$$6x = 6$$
$$x = 1$$

(b) $f'(1) = 0$, $f(1) = 5$

$3ax^2 + 2bx + 9 = 0$, $a(1)^3 + b(1)^2 + 9(1) + 1 = 5$

$3a(1)^2 + 2b(1) + 9 = 0$, $a + b = -5$

$3a + 2b = -9,$

$< - - a = -b - 5$

$3(-b - 5) + 2b = -9$

$-3b - 15 + 2b = -9$

$-b = 6$

$b = -6 - - >$, $a = -(-6) - 5$

$a = 1$

$\therefore y = x^3 - 6x^2 + 9x + 1$

As $a > 0$, local minimum at $x = 3$

$\therefore y = (3)^3 - 6(3)^2 + 9(3) + 1$

$\therefore y = 1$

$\therefore$ Local minimum $(3;1)$

(c) (1) $A(x) = A_{rec} - A_{bigcircle} - A_{smallcircle}$

$$A(x) = 2.4x - (2x)^2\pi - (\frac{4}{3}x)^2\pi$$

$$A(x) = 8x - 4x^2\pi - \frac{16}{9}x^2\pi$$

$$A(x) = 8x - \frac{52}{9}\pi x^2$$

(2) $At\ Max\ A'(x) = 0$

$$0 = 8 - \frac{104}{9}\pi x$$

$$\frac{104}{9}\pi x = 8$$

$$\therefore x = \frac{8.9}{104\pi}$$

$$\therefore x = 0{,}22m$$

QUESTION 11

$x^2 + 2x = k\left(x^2 - 2x + 2\right) + 2$

$x^2 + 2x - k\left(x^2 - 2x + 2\right) - 2 = 0$

$(1-k)x^2 + (2+2k)x + (-2k-2) = 0$

for distinct, realroots $\Delta > 0, b^2 - 4ac > 0$

$(2+2k)^2 - 4(1-k)(-2k-2) > 0$

$4 + 8k + 4k^2 - \left[(4-4k)(-2k-2)\right] > 0$

$4 + 8k + 4k^2 - \left[-8k - 8 + 8k^2 + 8k\right] > 0$

$4 + 8k + 4k^2 - 8k^2 + 8 > 0$

$-4k^2 + 8k + 12 > 0$

$k^2 - 2k - 3 < 0$

$\therefore -1 < k < 3,\ m \neq 1 \quad or \quad \{-1 < k < 1\} \cup \{-1 < k < 3\}$

QUESTION 12

(a) $\prod_{k=2}^{100}\left(1-\frac{1}{k}\right)=\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\ldots\ldots\left(1-\frac{1}{100}\right)$

$\prod_{k=2}^{100}\left(1-\frac{1}{k}\right)=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{4}{5}\ldots\ldots\frac{1}{89}.\frac{89}{99}.\frac{99}{100}.$

Note how consecutive denominator and numerator cancel each other.

$\prod_{k=2}^{100}\left(1-\frac{1}{k}\right)=\frac{1}{100}$

(b) Let *a, b, c* and *d* represent the four numbers.

$mean=\frac{a+b+c+d}{4}$

When the first number is added to the average of the other three numbers the result is 25.

Thus, $a+\frac{b+c+d}{3}=25$

Can be written as, $3a+b+c+d=75$

When the second number is added to the average of the other three numbers the result is 37.

Thus, $b+\frac{a+c+d}{3}=37$

Can be written as, $a+3b+c+d=111$

When the third number is added to the average of the other three numbers the result is 43.

Thus, $c+\frac{a+b+d}{3}=43$

Can be written as, $a+b+3c+d=129$

When the fourth number is added to the average of the other three numbers the result is 51.

Thus, $d+\frac{a+b+c}{3}=51$

Can be written as, $a+b+c+3d=153$

Adding (1), (2), (3) and (4), we obtain: $6a+6b+6c+6d=468$

Then dividing by we get $a+b+c+d=78$

$\therefore mean=\frac{a+b+c+d}{4}=\frac{78}{4}=19{,}5$

Total: 150 marks