INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
MEI 2024

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

(a) (1) $x^2 - 7x + 12 = 0$

$x = 3$ of $x = 4$

(2) $(5 - x) = 27^{\frac{2}{3}}$

$5 - x = 9$

$-x = 4$

$x = -4$

(3) $3x^2 - 11x + 6 \geq 0$

Faktoriseer

Metode

$x \leq \frac{2}{3}$ *or* $x \geq 3$

(4) $\log_b\left(\frac{2x+7}{x}\right) = \log_b 3^2$

$\frac{2x+7}{x} = 9$

$2x + 7 = 9x$

$-7x = -7$

$x = 1$

(5) $2^{x^2-6x} = 2^7$

$x^2 - 6x - 7 = 0$

$x = -1$ of $x = 7$

(b) $\Delta = 0$

$(k+2)^2 - 4(1)(4k - 7) = 0$

$k^2 + 4k + 4 - 16k + 28 = 0$

$k^2 - 12k + 32 = 0$

$k = 4$ of $k = 8$

VRAAG 2

(a) (1) $T_7 = 28$ $T_{12} = 73$

$28 = a + 6d$ $73 = a + 11d$

$a = 28 - 6d \;-->$ $73 = 28 - 6d + 11d$

$45 = 5d$

$<-- d = 9$

$a = 28 - 6d$

$a = -26$

(2) $S_{50} = \frac{50}{2}\left[2(-26) + 49(9)\right]$

$S_{50} = 9725$

(b) $T_8 = S_8 - S_7$

$T_8 = \frac{8}{2}\left[8(8)^2 + 7(8)\right] - \frac{7}{2}\left[8(7)^2 + 7(7)\right]$

$T_8 = 728{,}5$

(c) (1) $20 - a - a + 4 = 17 - 20 + a$

$24 - 2a = -3 + a$

$-3a = -27$

$a = 9$

(2) 4 ; 9 ; 20 ; 37

5 ; 11 ; 17

6 ; 6

$6 = 2a$ $3a + b = 5$ $a + b + c = 4$

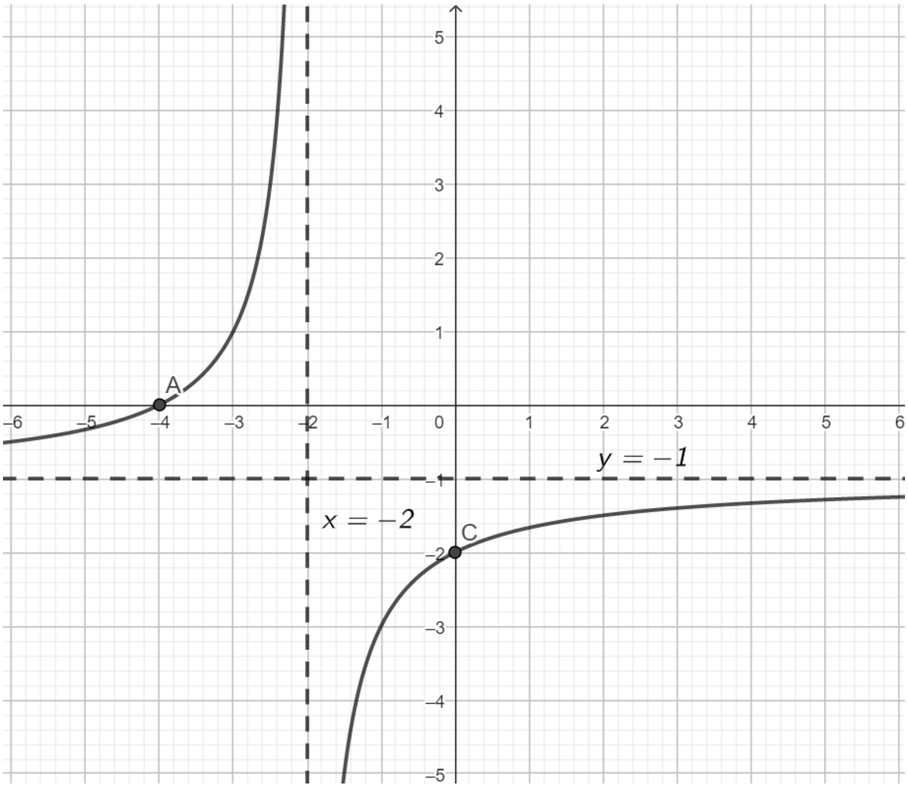
$a = 3$ $3(3) + b = 5$ $3 - 4 + c = 4$

$b = 5$ $c = 5$

$T_n = 3n^2 - 4n + 5$

VRAAG 3

(a)



(b) (1) $$x = \frac{-2}{y+1} - 2$$

$$x + 2 = \frac{-2}{y+1}$$

$$y + 1 = \frac{-2}{x+2}$$

$$y = \frac{-2}{x+2} - 1$$

(2) *Definisiegebied*: $x \in \mathbb{R}, x \neq -2$

VRAAG 4

(a) *Definisiegebied*: $x > -1$

Waardegebied: $y \geq -3$

(b) $\dfrac{\text{som van } x\text{-afsnitte}}{2} = 1$

$\dfrac{x_B + 4}{2} = 1$

$x_B = -2 \qquad B(-2;0)$

(c) *Gebruik* $(4;0)$ of $(-2;0)$.

$0 = p(4-1)^2 - 3$
$3 = 9p$

$p = \dfrac{1}{3}$

(d) $h(x) = -\left[\dfrac{1}{3}(x-1-2)^2 - 3 - 3\right]$

$h(x) = -\dfrac{1}{3}(x-3)^2 + 6$

(e) Vertikale asimptoot: $x = -1 \quad \therefore b = 1$

y-afsnit: $y = 1$

$f(x) = \log_a(x+1) - 2 \qquad$ *vervang* $(1;-3)$

$-3 = \log_a(1+1) - 2$
$-1 = \log_a(2)$

$a^{-1} = 2$

$a = \dfrac{1}{2}$

(f) by $f(x) \leq 0$:

$$0 = \log_{\frac{1}{2}}(x+1) - 2$$

$$2 = \log_{\frac{1}{2}}(x+1)$$

$$\left(\frac{1}{2}\right)^2 = x+1$$

$$x = -\frac{3}{4}$$

$$\therefore x \geq -\frac{3}{4}$$

VRAAG 5

(a) $$= \lim_{h \to 0} \left(\frac{1}{x^2 + 2xh + h^2} - \frac{1}{x^2} \right) \div h$$

$$= \lim_{h \to 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2\left(x^2 + 2xh + h^2\right)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{h\left(-2x - h\right)}{x^4 + 2x^3h + x^2h^2} \times \frac{1}{h}$$

$$= \frac{-2x - (0)}{x^4 + 2x^3(0) + x^2(0)^2}$$

$$= \frac{-2x}{x^4}$$

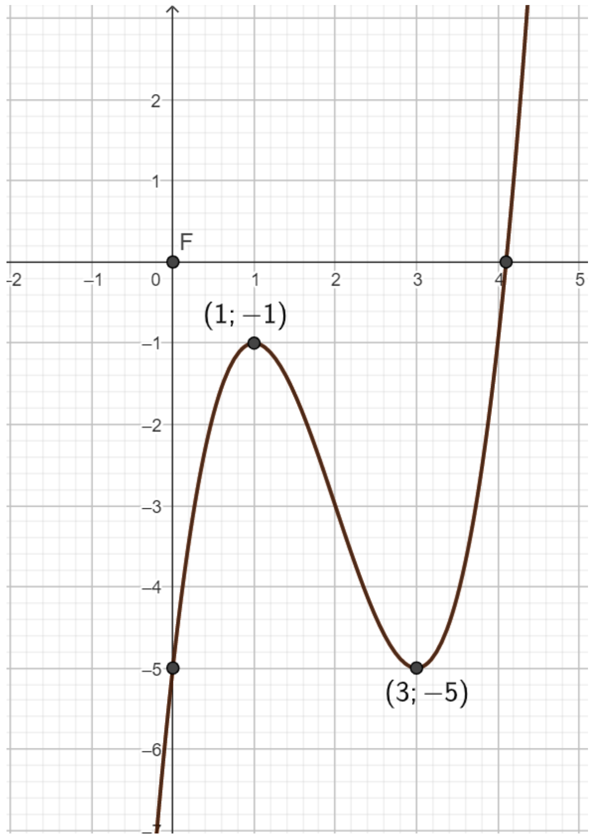
$$= \frac{-2}{x^3}$$

(b) $$y = \left(2x - 3x^{\frac{1}{2}}\right)^2$$

$$y = 4x^2 - 12x^{\frac{3}{2}} + 9x$$

$$\frac{dy}{dx} = 8x - 18x^{\frac{1}{2}} + 9 \quad \text{of} \quad \frac{dy}{dx} = 8x - 18\sqrt{x} + 9$$

(c) (1)



(2) Buigpunt is halfpad tussen draaipunte:

$$\frac{1+3}{2}=2$$

Konkaaf boontoe by $x>2$.

VRAAG 6

(a) $$F = 1\,600\left[\frac{\left(1+\frac{9\%}{12}\right)^{48}-1}{\frac{9\%}{12}}\right]$$

$$F = 92\,033{,}14$$

(b) $$131\,616{,}68 = 1\,600\left[\frac{\left(1+\frac{9\%}{12}\right)^{n}-1}{\frac{9\%}{12}}\right]+10\,000\left(1+\frac{9\%}{12}\right)^{12}$$

$$n = 60,\ldots$$
$$\therefore 61\ \textit{maande}$$

VRAAG 7

(a) 95% van R716 900

$= \text{R}681\ 055$

(b) $681\ 055 = x\left[\frac{1-\left(1+\frac{14{,}5\%}{12}\right)^{-72}}{\frac{14{,}5\%}{12}}\right]$

$x = 14\ 216{,}63$

(c) Uitstaande saldo (US) onmiddellik na die 54ste betaling:

$$US = 681\ 055\left(1+\frac{14{,}5\%}{12}\right)^{54} - 14\ 216{,}63\left[\frac{\left(1+\frac{14{,}5\%}{12}\right)^{54}-1}{\frac{14{,}5\%}{12}}\right]$$

$US = 228\ 748{,}11$

(d) $228\ 748{,}11 \times \frac{0{,}145}{12}$

$= \text{R}2\ 764{,}04$

VRAAG 8

(a) $y \leq 7$

$y \geq -\frac{1}{3}x + 4$ *or* $3y + x \geq 12$

$y \leq -\frac{5}{6}x + 10$ *or* $6y + 5x \leq 60$

(b) $4 \leq a \leq 7$ OF $a \in [4;7]$

$0 \leq b < 3$ of $b > 6$ OF $b \in [0;3)U(6;\infty)$

(c) (1) $S = 8x + 4y$

$y = -2x + \frac{S}{4}$

$S_{\text{min by } L} = 8(1) + 4(5)$

$S_{\text{min by } L} = 28$

(2) $T = 3y + 4x$

$y = -\frac{4}{3}x + \frac{T}{3}$

$T_{\text{maks by O}} = 3(5) + 4(6)$

$T_{\text{maks by O}} = 39$

(d) $S = cx + dy$ by $M(3;3)$ Maksimaliseer by gradiënte

$y = -\frac{c}{d}x + \frac{s}{d}$

$-1 < -\frac{c}{d} < -\frac{1}{3}$

$\frac{1}{3} < \frac{c}{d} < 1$

VRAAG 9

$\frac{4^q}{2^p} = \frac{1}{2}$ $\log_5\left(p^2 + q^2 - 4\right) = 2$

$2^{2q} = \frac{2^p}{2}$

$2^{2q+1} = 2^p$

$p = 2q + 1 ----->$ $\log_5\left((2q+1)^2 + q^2 - 4\right) = 2$

$\log_5\left(4q^2 + 4q + 1 + q^2 - 4\right) = 2\log_5 5$

$\log_5\left(5q^2 + 4q - 3\right) = \log_5 5^2$

$5q^2 + 4q - 3 = 25$

$5q^2 + 4q - 28 = 0$

$<----q = 2 \text{ or } q = -\frac{14}{5}$

vir $q = 2$; vir $q = -\frac{14}{5}$

$p = 2(2) + 1$; $p = 2\left(-\frac{14}{5}\right) + 1$

$p = 5$; $p = -\frac{23}{5}$

VRAAG 10

(a) Gebruik $T_n = ar^{n-1}$; $a = 5$, vir $n = 5$, 5^{de} sirkel

$$195{,}3125 = 5r^4$$

$$r^4 = \frac{625}{16}$$

$$r = \frac{5}{2}$$

(b) Gebruik die somformule vir n = 10 en tel bymekaar. Let daarop dat dit verdubbel moet word (middellynsom word benodig).

$$K_{\text{nuut}} = 2 \times \frac{a\left(r^n - 1\right)}{r - 1}$$

$$K_{\text{nuut}} = 2 \times \frac{3\left(\left(\frac{5}{2}\right)^{10} - 1\right)}{\frac{5}{2} - 1}$$

$$K_{\text{nuut}} = 38\,143{,}00$$

(c) Patroonuitdrukking vir oppervlakte:

$$\text{Oppervlakte} = \pi \cdot 5^2 + \pi \cdot \left[\left(\frac{5}{2}\right) \cdot 5\right]^2 + \pi \cdot \left[\left(\frac{5}{2}\right)^2 \cdot 5\right]^2 + \ldots.. + \pi \cdot \left[\left(\frac{5}{2}\right)^9 \cdot 5\right]^2$$

$$\text{Oppervlakte} = \pi \cdot 5^2 \left[1 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^4 + \left(\frac{5}{2}\right)^6 + \ldots. + \left(\frac{5}{2}\right)^{18}\right]$$

$\Rightarrow [\_]$ 10 terme wat meetkundige reeks vorm met $r = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$$\text{Oppervlakte} = 25\pi \times \frac{1\left[\left(\frac{25}{4}\right)^{10} - 1\right]}{\frac{25}{4} - 1}$$

$$\text{Oppervlakte} = 433\ 092\ 710\pi \text{ eenhede}^2$$

VRAAG 11

(a) (1) $f(x)=\dfrac{bx^{-1}}{3}+\dfrac{2x^{\frac{1}{2}}}{3}$

$$f'(x)=-\frac{bx^{-2}}{3}+\frac{x^{-\frac{1}{2}}}{3}$$

$$f'\left(\frac{1}{4}\right)=-\frac{b\left(\frac{1}{4}\right)^{-2}}{3}+\frac{\left(\frac{1}{4}\right)^{-\frac{1}{2}}}{3}$$

$$f'\left(\frac{1}{4}\right)=\frac{-16b+2}{3}$$

(2) $y=-10x+\dfrac{4}{3}$

$$\textit{gradiënt}=-10$$

$$\frac{-16b+2}{3}=-10$$

$$b=2$$

$$f(x)=\frac{2+2x\sqrt{x}}{3}$$

$$f\left(\frac{1}{4}\right)=\frac{2+2\left(\frac{1}{4}\right)\sqrt{\left(\frac{1}{4}\right)}}{3}=\frac{3}{4}$$

$$y=-10x+c \quad \textit{vervang} \ \left(\frac{1}{4};\frac{3}{4}\right)$$

$$\frac{3}{4}=-10\left(\frac{1}{4}\right)+c$$

$$c=\frac{13}{4}$$

$$\therefore y=-10x+\frac{13}{4}$$

(b) $y^2 + y^2 = 4x^2$ *Pythagoras*

$2y^2 = 4x^2$

$y = \sqrt{2}x$

Oppervlakte van $\Delta = \frac{1}{2}y^2 = \frac{1}{2}\left(\sqrt{2}x\right)^2 = \frac{1}{2}\cdot 2x^2 = x^2$

vir volume

$x^2h = 64$

$h = \frac{64}{x^2}$

vir buiteoppervlakte van prisma

$A = 2x^2 + 2yh$

$A(x) = 2x^2 + 2\cdot\sqrt{2}x\cdot\frac{64}{x^2}$

$A(x) = 2x^2 + \frac{128\sqrt{2}}{x}$

by min $A'(x) = 0$

$0 = x - 128\sqrt{2}x^{-2}$

$x^3 = 128\sqrt{2}$

$x = 5{,}7$ *eenhede*

Minimum oppervlakte

$A(5{,}7) = 2(5{,}7)^2 + \frac{128\sqrt{2}}{5{,}7} = 96{,}7$ *eenhede*^{2}

VRAAG 12

In 1 uur: X en Y vul $\frac{1}{12}$ van die tenk

X en Z vul $\frac{1}{15}$ van die tenk

Y en Z vul $\frac{1}{20}$ van die tenk

Optelvergelyking: $X + Y + X + Z + Y + Z = \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$

$$2(X + Y + Z) = \frac{1}{5}$$

X, Y en Z saam vul $\frac{1}{10}$ van die tenk.

Bygevolg, Y alleen vul $\frac{1}{10} - \frac{1}{15} = \frac{1}{30}$ van die tenk in 1 uur

Dus, 30 uur om die tenk vol te maak.

Totaal: 150 punte