INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2023

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

(a) (1) $x = 2$ or $x = -3$

(2) (i) $x = 3$

(ii) $x = -\sqrt{5}$ or $x = \sqrt{5}$

(iii) $x = -\frac{1}{2}$

(3) $x^2 - 3x + 2 > 0$
$cv's: 1; 2$
$x < 1$ or $x > 2$

(4) $x - 5 = \sqrt{x - 3}$
$x^2 - 10x + 25 = x - 3$
$x^2 - 11x + 28 = 0$
$(x - 7)(x - 4) = 0$
$x = 7$ or $x \neq 4$
valid invalid

(5) $\log_3 2(x+5) = \log_3 16$
$2(x+5) = 16$
$2x + 10 = 16$
$2x = 6$
$x = 3$ (3)

(b) $\Delta = 0$
$(-2)^2 - 4(-1)(2+k) = 0$
$4 - 8 - 4k = 0$
$4k = -4$
$k = -1$

QUESTION 2

(a) $57 = 120 + (10 - 1)d \quad 57 = 120 + (10 - 1)d$

$9d = -63$

$d = -7$

$T_{15} = 120 + 14(-7)$

$T_{15} = 22$

(b) $\frac{10k + 2}{5k + 1} = \frac{5k + 1}{2k + 2}$

$(10k + 2)(2k + 2) = (5k + 1)^2$

$20k^2 + 24k + 4 = 25k^2 + 10k + 1$

$0 = 5k^2 - 14k - 3$

$k \neq -\frac{1}{5}$ or $k = 3$

(c) (1) 2nd diff makes an arithmetic sequence with $d = 6$

Thus we are looking T_{11} .

$T_n = a + (n - 1)d$

$T_{11} = 7 + 6(10)$

$T_{11} = 67$

(2) now : is a quadractic sequence

T_1 ; T_2 ; T_3 ; T_4

7 ; 13 ; 19 ; ...

6 ; 6

$2a = 6$ $\quad$ $3a + b = 7$

$a = 3$ $\quad$ $a + b = 7$

$b = -2$

$\therefore T_n = 3n^2 - 2n + c$

now :

$T_{89} = 23\,594$

$\therefore 3(89)^2 - 2(89) + c = 23\,594$

$\therefore c = 9$

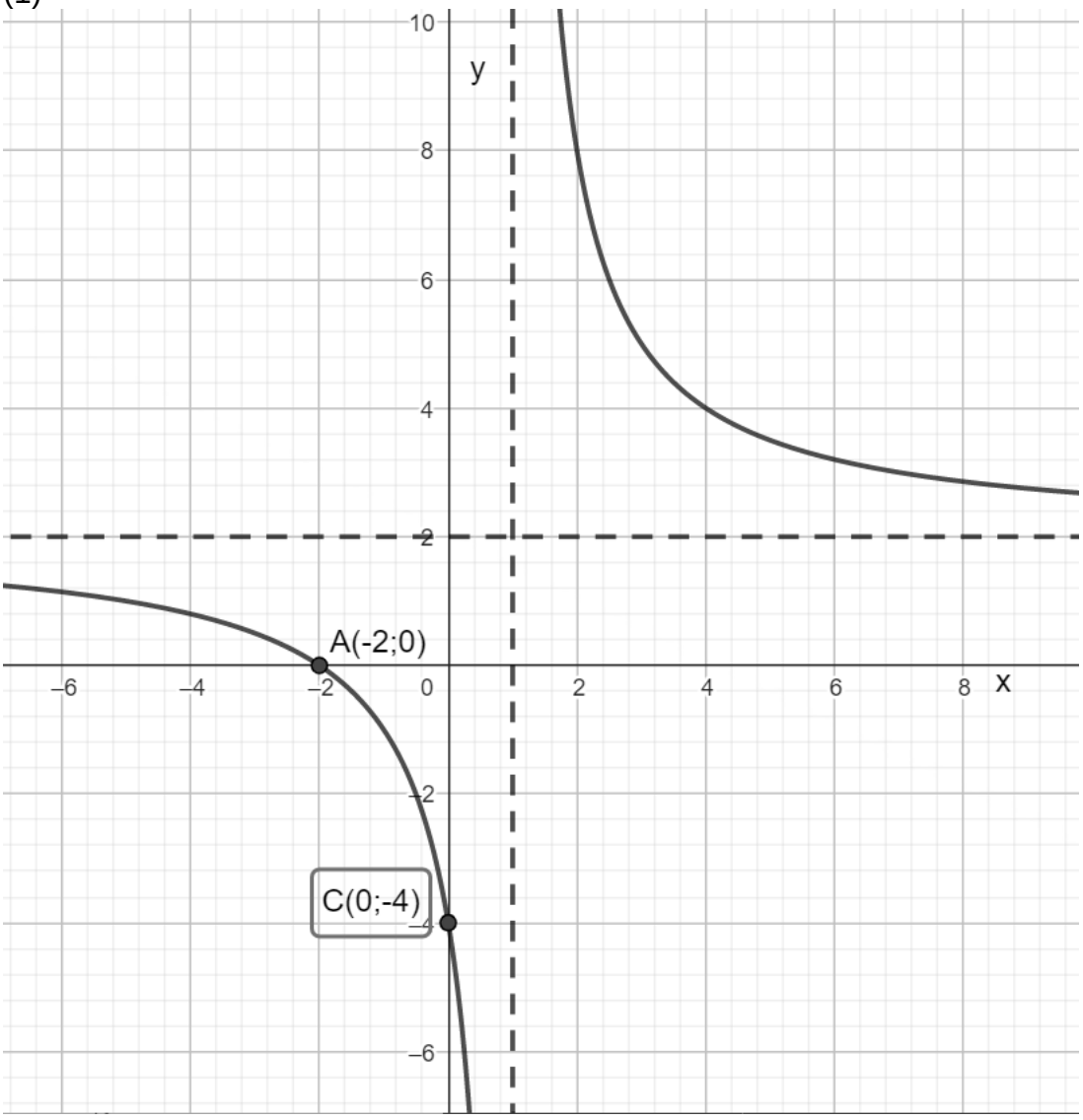
$\therefore T_n = 3n^2 - 2n + 9$

$\therefore T_{69} = 3(69)^2 - 2(69) + 9$

$\therefore T_{69} = 14\,154$

QUESTION 3

(a) (1)



(2) $x \in R\,;\, x \neq 1$

(3) $y = x + 1$

(b) (1) $x = 3^{y+1}$

$y + 1 = \log_3 x$

$y = \log_3 (x) - 1$

(2) (i) $x \in R$

(ii) $0 < x < 3$

QUESTION 4

(a) $y = 2^0 - 4$

$y = -3$

$\therefore C(0;-3)$

$0 = 2^x - 4$

$2^2 = 2^x$

$x = 2$

$\therefore B(2;0)$

(b) $g(x) = a(x-2)^2 \; sub(0;2)$

$2 = a(4)$

$\therefore a = \frac{1}{2}$

$g(x) = \frac{1}{2}(x-2)^2$

$g(x) = \frac{1}{2}x^2 - 2x + 2$

$b = -2$

$c = 2$

(c) $-f(x) =$

$h(x) = -(2^x - 4)$

$h(x) = -2^x + 4$

(d) $x \geq 0$

QUESTION 5

(a) $f(x+h) = 3(x+h)^2 + 2(x+h)$

$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$

$$f'(x) = \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 3x^2 - 2x}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(6x + 3h + 2)}{h}$$

$f'(x) = 6x + 3(0) + 2$

$f'(x) = 6x + 2$

(b) (1) $y = x^3 - x$

$$\frac{dy}{dx} = 3x^2 - 1$$

(2) $$f(x) = \frac{x^3}{2} - \frac{1}{2} - x^{\frac{3}{2}}$$

$$f'(x) = \frac{3}{2}x^2 - \frac{3}{2}x^{\frac{1}{2}}$$

(c) (1) $-2 < x < 1$ or $x > 3$

(2) $-0,79 < x < 2,12$

QUESTION 6

(a) $$F = 3\,500\left[\frac{\left(1+\frac{12\%}{12}\right)^{38}-1}{\frac{12\%}{12}}\right]$$

$$F = \text{R}160\ 834{,}53$$

(b) $$285\ 843{,}84 = 3\,500\left[\frac{\left(1+\frac{12\%}{12}\right)^{n}-1}{\frac{12\%}{12}}\right]$$

$$0{,}81669\ldots = 1{,}01^{n}-1$$

$$1{,}81669\ldots = 1{,}01^{n}$$

$$\therefore n = \log_{1{,}01} 1{,}81669\ldots$$

$$\therefore n \approx 59{,}99$$

60 months

QUESTION 7

(a) $90\%\text{ of R}1\,800\,000$
$= \text{R}1\,620\,000$

(b) $$1\,620\,000 = x\left[\frac{1-\left(1+\frac{8{,}5\%}{12}\right)^{-240}}{\frac{8{,}5\%}{12}}\right]$$
$\therefore x = 14\,058{,}74$

(c) OB payments left 82
$$OB = 14\,058{,}74\left[\frac{1-\left(1+\frac{8{,}5\%}{12}\right)^{-82}}{\frac{8{,}5\%}{12}}\right]$$
$OB = 872\,147{,}31$

or
$$OB = 1\,620\,000\left(1+\frac{8{,}5\%}{12}\right)^{158} - 14\,058{,}74\left[\frac{\left(1+\frac{8{,}5\%}{12}\right)^{158}-1}{\frac{8{,}5\%}{12}}\right]$$
$OB = 872\,147{,}31$

(d) $OB = 872\,147{,}31$
Capital reduced by:
$879\,972{,}91 - 872\,147{,}31 = 7\,825{,}59$

Share of payment reducing capital
$$= \frac{7\,825{,}59}{14\,058{,}74} \times 100$$
$$= 55{,}66\%$$

QUESTION 8

(a) Distance (D) $= 10 + 15 + 15\left(\frac{3}{4}\right) + 15\left(\frac{3}{4}\right)^2 + 15\left(\frac{3}{4}\right)^3 + \ldots$

form a geometric series after 10

$a = 15$ and $r = \frac{3}{4}$

$D = 10 + S_\infty$

$D = 10 + \frac{15}{1 - \frac{3}{4}}$

$D = 70 \text{ m}$

(b) $\left\{120 + 80 + \frac{160}{3} + \ldots\right\} + \left\{3 + 6 + 12 + 24 + \ldots\right\} = 786\ 789$

$\left\{r = \frac{2}{3}\ ;\ a = 120\right\} + \left\{r = 2\ ;\ a = 3\right\} = 786\ 789$

$\left\{\frac{120}{1 - \frac{2}{3}}\right\} + \frac{3\left(2^n - 1\right)}{2 - 1} = 786\ 789$

$360 + 3\left(2^n - 1\right) = 786\ 789$

$3\left(2^n - 1\right) = 786\ 429$

$2^n - 1 = 262\ 143$

$2^n = 262\ 144$

$2^n = 2^{18}$

$\therefore n = 18$

QUESTION 9

(a) $h'(x) = 3x^2 + 2px + q$

$h''(x) = 6x + 2p$

at point of inflection $h''(x) = 0$

$6(2) + 2p = 0$

$2p = -12$

$p = -6$

$-16 = (2)^3 + p(2)^2 + q(2) + 8$

sub $p = -6$

$-32 = 4(-6) + 2q$

$2q = -8$

$q = -4$

$\therefore p = -6; \quad q = -4$

(b) (1) $x^3 - 9x^2 + 14x + 29 = 2x^2 - 6x - 3$

$x^3 - 11x^2 + 20x + 32 = 0$

$(x + 1)(x - 4)(x - 8) = 0$

$r = -1; \quad s = 4; \quad t = 8$

(2) $AB = x^3 - 9x^2 + 14x + 29 - (2x^2 - 6x - 3)$

$AB = x^3 - 11x^2 + 20x + 32$

$\frac{dAB}{dx} = 3x^2 - 22x + 20$

at extremum $\frac{dAB}{dx} = 0$

$3x^2 - 22x + 20 = 0$

$x = 6{,}3$ or $x = 1{,}1$

clearly max at $x = 1{,}1$ since it must lie $-1 < x < 4$

so max length of $AB = (1{,}1)^3 - 11(1{,}1)^2 + 20(1{,}1) + 32$

so max length of $AB = 42{,}0$ units

QUESTION 10

(a) $y \geq 15$

$x \leq 40$

$x \geq 10$

$y \leq -x + 60$

$y \leq -\frac{5}{8}x + 50$

(b) Let product $A = x$ and product $B = y$

$P = 7x + 5y$

$5y = P - 7x$

$y = \frac{P}{5} - \frac{7}{5}x$

Max Profit at point $\mathrm{R}(40\,;20)$

$P_{\max} = 7(40) + 5(20)$

$P_{\text{maks}} = 380$

(c) to maximise at Q

the gradient $-\frac{k}{5}$ must lie

below the gradients of the

line containing Q

$-1 < m < -\frac{5}{6}$

$-1 < -\frac{k}{5} < -\frac{5}{6}$

$-5 < -k < -\frac{25}{6}$

$\frac{25}{6} < k < 5$

QUESTION 11

$3^y \cdot 9^x = \frac{27^x}{3} \ldots eqn1$

$3^{y+2x} = 3^{3x-1}$

$\therefore y + 2x = 3x - 1$

$\therefore x = y + 1$

now sub into *eqn*2

$3x + y + 4x + y = 16$

$7x + 2y = 16 \ldots eqn2$

$\therefore 7(y+1) + 2y = 16$

$9y = 9$

$y = 1$

now sub y into x

$x = 1 + 1$

$x = 2$

$\therefore x = 2 ; y = 1$

QUESTION 12

(a) $f(0) = 2023$

$$f(1) = \frac{1+f(0)}{1-f(0)} = \frac{1+2023}{1-2023} = -\frac{1012}{1011}$$

$$f(2) = \frac{1+f(1)}{1-f(1)} = \frac{1+-\frac{1012}{1011}}{1--\frac{1012}{1011}} = -\frac{1}{2023}$$

$$f(3) = \frac{1+f(2)}{1-f(2)} = \frac{1+-\frac{1}{2023}}{1--\frac{1}{2023}} = \frac{1012}{1011}$$

$$f(4) = \frac{1+f(3)}{1-f(3)} = \frac{1+\frac{1012}{1011}}{1-\frac{1012}{1011}} = 2023$$

$\therefore$ cycles every 4 times

$$\therefore f(2022) = -\frac{1012}{1011}$$ (5)

(b) $Speed = \frac{\text{Distance}}{Time}$

	Distance	Speed	Time
Alice	x	A	$\frac{x}{A}$
Ben	x	B	$\frac{x}{B}$
Chad	x	C	$\frac{x}{C}$

when Alice finshes : Ben's Distance : $x-17=\frac{x}{A}\times B$ ...(1)

Chad's Distance : $x-37=\frac{x}{A}\times C$ ...(2)

when Ben finshes : Chad's Distance : $x-24=\frac{x}{B}\times C$ ...(3)

Divide...(2) $\div$ ...(3) : $\frac{x-37}{x-24}=\frac{xc}{A}\div\frac{xc}{B}=\frac{B}{A}$

now sub in ...(1)

$\therefore x-17=x\left(\frac{x-37}{x-24}\right)$

$\therefore (x-17)(x-24)=x(x-37)$

$\therefore x^2-41x+408=x^2-37x$

$\therefore 408=4x$

$\therefore x=102$

The length of the race is 102 metres

Total: 150 Marks