INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
MEI 2023

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

IEB Copyright © 2023

VRAAG 1

(a) (1) $x = 2$ of $x = -3$

(2) (i) $x = 3$

(ii) $x = -\sqrt{5}$ of $x = \sqrt{5}$

(iii) $x = -\dfrac{1}{2}$

(3) $x^2 - 3x + 2 > 0$
kritieke waardes: $1; 2$
$x < 1$ of $x > 2$

(4) $x - 5 = \sqrt{x-3}$
$x^2 - 10x + 25 = x - 3$
$x^2 - 11x + 28 = 0$
$(x-7)(x-4) = 0$
$x = 7$ of $x \neq 4$
geldig ongeldig

(5) $\log_3 2(x+5) = \log_3 16$
$2(x+5) = 16$
$2x + 10 = 16$
$2x = 6$
$x = 3$

(b) $\Delta = 0$
$(-2)^2 - 4(-1)(2+k) = 0$
$4 - 8 - 4k = 0$
$4k = -4$
$k = -1$

VRAAG 2

(a) $57 = 120 + (10-1)d$

$9d = -63$

$d = -7$

$T_{15} = 120 + 14(-7)$

$T_{15} = 22$

(b) $\dfrac{10k+2}{5k+1} = \dfrac{5k+1}{2k+2}$

$(10k+2)(2k+2) = (5k+1)^2$

$20k^2 + 24k + 4 = 25k^2 + 10k + 1$

$0 = 5k^2 - 14k - 3$

$k \neq -\dfrac{1}{5}$ of $k = 3$

(c) (1) 2^{de} verskil vorm 'n rekenkundige ry met $d = 6$.
Ons soek dus na T_{11}.

$T_n = a + (n-1)d$

$T_{11} = 7 + 6(10)$

$T_{11} = 67$

(2) nou : kwadratiese ry

T_1 ; T_2 ; T_3 ; T_4

7 ; 13 ; 19 ; ...

6 ; 6

$2a = 6$ $\quad$ $3a + b = 7$

$a = 3$ $\quad$ $a + b = 7$

$b = -2$

$\therefore T_n = 3n^2 - 2n + c$

nou:

$T_{89} = 23\,594$

$\therefore 3(89)^2 - 2(89) + c = 23\,594$

$\therefore c = 9$

$\therefore T_n = 3n^2 - 2n + 9$

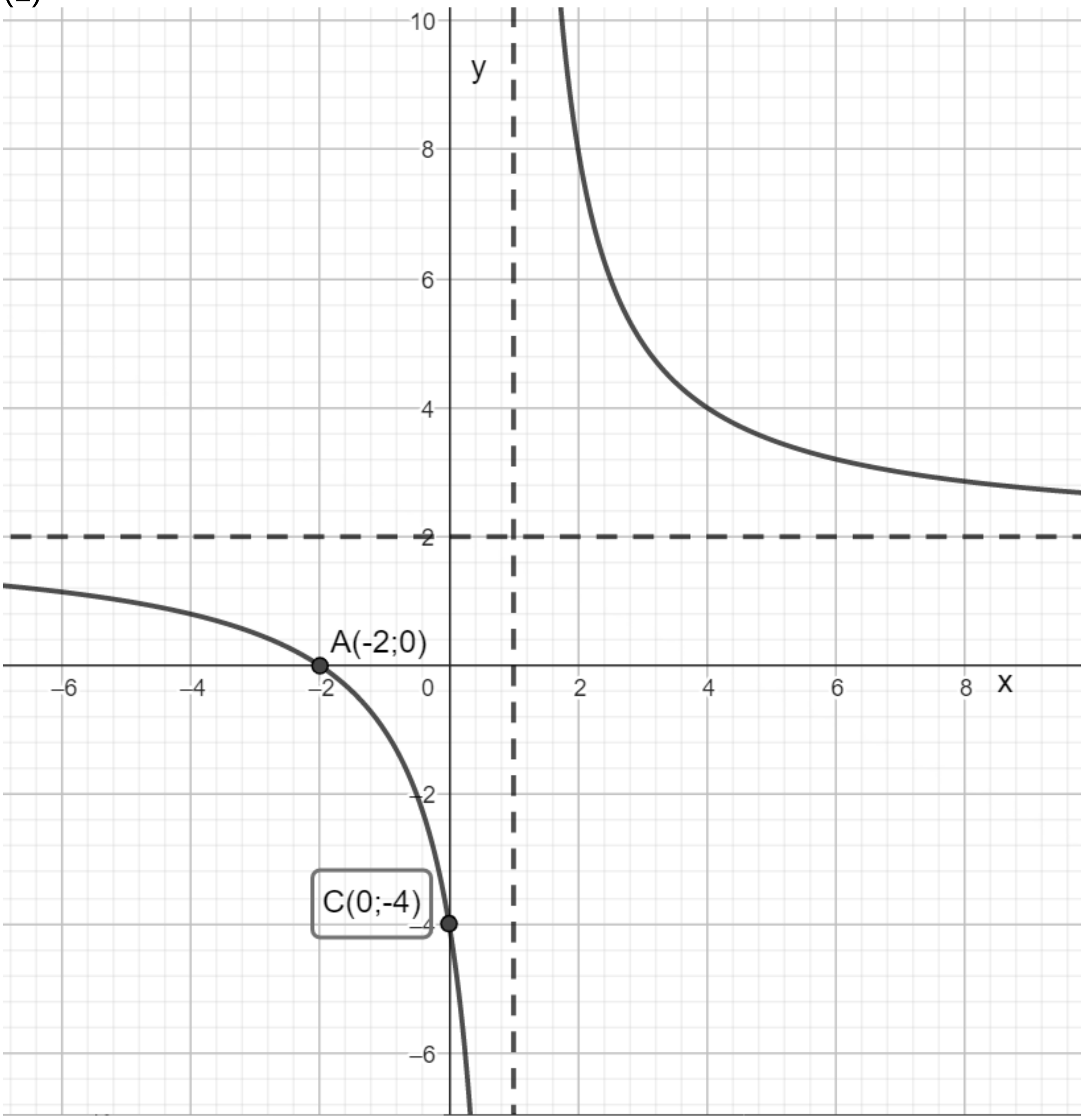
$\therefore T_{69} = 3(69)^2 - 2(69) + 9$

$\therefore T_{69} = 14\,154$

IEB Copyright © 2023

VRAAG 3

(a) (1)



(2) $x \in R$; $x \neq 1$

(3) $y = x + 1$

(b) (1) $x = 3^{y+1}$

$y + 1 = \log_3 x$

$y = \log_3 (x) - 1$

(2) (i) $x \in R$

(ii) $0 < x < 3$

VRAAG 4

(a) $y = 2^0 - 4$

$y = -3$

$\therefore C(0\,;-3)$

$0 = 2^x - 4$

$2^2 = 2^x$

$x = 2$

$\therefore B(2\,;0)$

(b) $g(x) = a(x-2)^2$ vervang $(0\,;2)$

$2 = a(4)$

$\therefore a = \frac{1}{2}$

$g(x) = \frac{1}{2}(x-2)^2$

$g(x) = \frac{1}{2}x^2 - 2x + 2 +$

$b = -2$

$c = 2$

(c) $-f(x) =$

$h(x) = -(2^x - 4)$

$h(x) = -2^x + 4$

(d) $x \geq 0$

VRAAG 5

(a) $f(x+h) = 3(x+h)^2 + 2(x+h)$

$f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h$

$f'(x) = \lim\limits_{h\to 0} \dfrac{3x^2 + 6xh + 3h^2 + 2x + 2h - 3x^2 - 2x}{h}$

$f'(x) = \lim\limits_{h\to 0} \dfrac{6xh + 3h^2 + 2h}{h}$

$f'(x) = \lim\limits_{h\to 0} \dfrac{h(6x + 3h + 2)}{h}$

$f'(x) = 6x + 3(0) + 2$

$f'(x) = 6x + 2$

(b) (1) $y = x^3 - x$

$\dfrac{dy}{dx} = 3x^2 - 1$

(2) $f(x) = \dfrac{x^3}{2} - \dfrac{1}{2} - x^{\frac{3}{2}}$

$f'(x) = \dfrac{3}{2}x^2 - \dfrac{3}{2}x^{\frac{1}{2}}$

(c) (1) $-2 < x < 1$ of $x > 3$

(2) $-0{,}79 < x < 2{,}12$

IEB Copyright © 2023

VRAAG 6

(a) $$F = 3\,500\left[\frac{\left(1+\frac{12\%}{12}\right)^{38}-1}{\frac{12\%}{12}}\right]$$

$$F = \text{R}160\ 834{,}53$$

(b) $$285\ 843{,}84 = 3\,500\left[\frac{\left(1+\frac{12\%}{12}\right)^{n}-1}{\frac{12\%}{12}}\right]$$

$$0{,}81669\ldots = 1{,}01^{n}-1$$

$$1{,}81669\ldots = 1{,}01^{n}$$

$$\therefore n = \log_{1{,}01} 1{,}81669\ldots$$

$$\therefore n \approx 59{,}99$$

60 maande

VRAAG 7

(a) 90% van $R1\,800\ 000$
$= R1\,620\ 000$

(b) $$1\,620\,000 = x\left[\frac{1-\left(1+\frac{8,5\%}{12}\right)^{-240}}{\frac{8,5\%}{12}}\right]$$
$\therefore x = 14\,058,74$

(c) Uitstaande saldo: 82 betalings oor
$$US = 14\ 058,74\left[\frac{1-\left(1+\frac{8,5\%}{12}\right)^{-82}}{\frac{8,5\%}{12}}\right]$$
$US = 872\ 147,31$

of

$$US = 1\,620\ 000\left(1+\frac{8,5\%}{12}\right)^{158} - 14\ 058,74\left[\frac{\left(1+\frac{8,5\%}{12}\right)^{158}-1}{\frac{8,5\%}{12}}\right]$$
$US = 872\ 147,31$

(d) $US = 872\ 147,31$
Kapitaal verminder met:
$879\ 972,91 - 872\,147,31 = 7\ 825,59$

Gedeelte van betaling wat kapitaal verminder
$= \frac{7\ 825,59}{14\ 058,74}\times 100$
$= 55,66\%$

VRAAG 8

(a) Afstand (D) $= 10 + 15 + 15\left(\frac{3}{4}\right) + 15\left(\frac{3}{4}\right)^2 + 15\left(\frac{3}{4}\right)^3 + ...$

vorm meetkundige reeks na 10

$a = 15$ en $r = \frac{3}{4}$

$D = 10 + S_\infty$

$D = 10 + \dfrac{15}{1 - \frac{3}{4}}$

$D = 70$ m

(b) $\left\{120 + 80 + \frac{160}{3} + ...\right\} + \{3 + 6 + 12 + 24 + ...\} = 786\ 789$

$\left\{r = \frac{2}{3}\ ;\ a = 120\right\} + \{r = 2\ ;\ a = 3\} = 786\ 789$

$\left\{\dfrac{120}{1 - \frac{2}{3}}\right\} + \dfrac{3\left(2^n - 1\right)}{2 - 1} = 786\ 789$

$360 + 3\left(2^n - 1\right) = 786\ 789$

$3\left(2^n - 1\right) = 786\ 429$

$2^n - 1 = 262\ 143$

$2^n = 262\ 144$

$2^n = 2^{18}$

$\therefore n = 18$ ✓

VRAAG 9

(a) $h'(x) = 3x^2 + 2px + q$

$h''(x) = 6x + 2p$

by buigpunt $h''(x) = 0$

$6(2) + 2p = 0$

$2p = -12$

$p = -6$

$-16 = (2)^3 + p(2)^2 + q(2) + 8$

vervang $p = -6$

$-32 = 4(-6) + 2q$

$2q = -8$

$q = -4$

$\therefore p = -6; \quad q = -4$

(b) (1) $x^3 - 9x^2 + 14x + 29 = 2x^2 - 6x - 3$

$x^3 - 11x^2 + 20x + 32 = 0$

$(x+1)(x-4)(x-8) = 0$

$r = -1; \quad \text{s} = 4; \quad \text{t} = 8$

(2) $AB = x^3 - 9x^2 + 14x + 29 - (2x^2 - 6x - 3)$

$AB = x^3 - 11x^2 + 20x + 32$

$\dfrac{dAB}{dx} = 3x^2 - 22x + 20$

by ekstremum $\dfrac{dAB}{dx} = 0$

$3x^2 - 22x + 20 = 0$

$x = 6{,}3$ of $x = 1{,}1$

duidelik maks by $x = 1{,}1$ want dit moet lê: $-1 < x < 4$

dus maks lengte van $AB = (1{,}1)^3 - 11(1{,}1)^2 + 20(1{,}1) + 32$

dus maks lengte van $AB = 42{,}0$ eenhede

VRAAG 10

(a) $y \geq 15$

$x \leq 40$

$x \geq 10$

$y \leq -x + 60$

$y \leq -\frac{5}{8}x + 50$

(b) Laat produk $A = x$ en produk $B = y$

$P = 7x + 5y$

$5y = P - 7x$

$y = \frac{P}{5} - \frac{7}{5}x$

Maks wins by punt $\text{R}(40\,;20)$

$P_{\text{maks}} = 7(40) + 5(20)$

$P_{\text{maks}} = 380$ (4)

(c) om te maksimaliseer by Q
die gradiënt $-\frac{k}{5}$ moet lê
onder die gradiënte van die
lyne wat Q bevat

$-1 < m < -\frac{5}{6}$

$-1 < -\frac{k}{5} < -\frac{5}{6}$

$-5 < -k < -\frac{25}{6}$

$\frac{25}{6} < k < 5$

VRAAG 11

$3^y \cdot 9^x = \dfrac{27^x}{3}$ ... verg 1

$3^{y+2x} = 3^{3x-1}$

$\therefore y + 2x = 3x - 1$

$\therefore x = y + 1$

vervang nou in verg 2

$3x + y + 4x + y = 16$

$7x + 2y = 16$ ... verg 2

$\therefore 7(y+1) + 2y = 16$

$9y = 9$

$y = 1$

vervang nou y in x

$x = 1 + 1$

$x = 2$

$\therefore x = 2\,;\, y = 1$

VRAAG 12

(a) $f(0) = 2023$

$$f(1) = \frac{1+f(0)}{1-f(0)} = \frac{1+2023}{1-2023} = -\frac{1012}{1011} \checkmark$$

$$f(2) = \frac{1+f(1)}{1-f(1)} = \frac{1+-\frac{1012}{1011}}{1--\frac{1012}{1011}} = -\frac{1}{2023} \checkmark$$

$$f(3) = \frac{1+f(2)}{1-f(2)} = \frac{1+-\frac{1}{2023}}{1--\frac{1}{2023}} = \frac{1012}{1011} \checkmark$$

$$f(4) = \frac{1+f(3)}{1-f(3)} = \frac{1+\frac{1012}{1011}}{1-\frac{1012}{1011}} = 2023 \checkmark$$

$\therefore$ sikliseer elke 4 keer

$$\therefore f(2022) = -\frac{1012}{1011}$$

(b) $\text{Spoed} = \dfrac{\text{Afstand}}{\text{Tyd}}$

	Afstand	Spoed	Tyd
Alice	x	A	$\frac{x}{A}$
Ben	x	B	$\frac{x}{B}$
Chad	x	C	$\frac{x}{C}$

wanneer Alice klaar is: Ben se afstand: $x-17 = \dfrac{x}{A}\times B$ ...(1)

Chad se afstand: $x-37 = \dfrac{x}{A}\times C$ ...(2)

wanneer Ben klaar is: Chad se afstand: $x-24 = \dfrac{x}{B}\times C$ ...(3)

Deel ...(2) $\div$ ...(3) : $\dfrac{x-37}{x-24} = \dfrac{xc}{A} \div \dfrac{xc}{B} = \dfrac{B}{A}$

vervang nou in ...(1)

$\therefore x-17 = x\left(\dfrac{x-37}{x-24}\right)$

$\therefore (x-17)(x-24) = x(x-37)$

$\therefore x^2 - 41x + 408 = x^2 - 37x$

$\therefore 408 = 4x$

$\therefore x = 102$

Die lengte van die wedloop is 102 meter.

Totaal: 150 punte

IEB Copyright © 2023