INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) $2^x + 2.2^x + 2^2.2^x$
$= 7.2^x$

(b) $x = -8$ or $x = -2$
$x^2 \neq 8$

(c) $\sqrt{y+10} = y-2$
$\therefore y+10 = y^2 - 4y + 4$
$0 = y^2 - 5y - 6$
$0 = (y-6)(y+1)$

$x(x+1) = 6$	$x(x+1) = -1$	OR they could eliminate before
$x^2 + x - 6 = 0$	$x^2 + x + 1 = 0$	they substitute back $y \neq -1$
$(x+3)(x-2) = 0$	n/a	
$x = -3,\ x = 2$		

Students who kept in terms of x, squared both sides and arrived at the correct quartic were awarded one mark. Completely correct solution in terms of x would have earned 5 marks.

QUESTION 2

(a) $\log 7{,}5$
$\log 15 - \log 2$ or any reasonable equivalent
= $\log 5 + \log 3 - \log 2$ or any reasonable equivalent/ca
$= \log 10 - \log 2 + \log 3 - \log 2$ or any reasonable equivalent/ca
$= b - 2a + 1$

(b) $\log_2(x+1) + \log_2(x-5) = \log_2 7$
$\therefore \log_2(x+1)(x-5) = \log_2 7$
$x^2 - 4x - 5 = 7$
$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$
$x = 6; x \neq -2$

(c) $\log(y-1) = \log x^2 + \log 4$
$\therefore \log(y-1) = \log 4x^2$
$\therefore y - 1 = 4x^2$
$\therefore y = 4x^2 + 1$

QUESTION 3

(a) $41167{,}14 = P\left(1+\frac{0{,}15}{12}\right)^{84}$

$\therefore D = \text{R}14\,500{,}00$

(b) (i) R450 000

(ii) reducing balance

(iii) $302\,580 = 480\,000(1-i)^2$

$\therefore i = 0{,}18$ or 18%

Marks awarded for correct A and P, correct n, formula, accurate answer

(iv) $x = 450\,000(1-0{,}18)^7$

$= \text{R}112\,178{,}46$

QUESTION 4

(a) $f'(x) = \lim\limits_{h\to 0}\frac{-3(x+h)^2+4+3(x^2-4)}{h}$

One mark formula, one mark correct sub

$= \lim\limits_{h\to 0}\frac{-6xh-3h^2}{h}$

$= \lim\limits_{h\to 0}(-6x-3h)$

$= -6x$

(b) (i) $f(x) = \frac{1}{2}x^2 - 3x^{-3}$

$f'(x) = x + \frac{9}{x^4}$

$\therefore f'(-1) = -1+9$

$= 8$

(ii) $f(-1) = \frac{1}{2}(-1)^2 - \frac{3}{(-1)^3}$

$= \frac{7}{2}$

$\therefore y - \frac{7}{2} = 8(x+1)$

$y = 8x + \frac{23}{2}$

$2y = 16x + 23$

$16x - 2y = -23$

QUESTION 5

(a) $f(x) = -\left(x^2 - 4x - 10\right)$

$= -\left[(x-2)^2 - 4 - 10\right]$

$= -(x-2)^2 + 14$

(b) $k = 2$ Accept x, and > =

(c) $x = -(y-2)^2 + 14$

$\therefore f^{-1}(x) = 2 + \sqrt{14 - x}$

QUESTION 6

(a) $T_3 - T_2 = T_2 - T_1$

$2(x-3) = 2x - 4 + 8 - 2x$

$\therefore x = 5$

$\therefore a = 6 \qquad d = -4$

$S_{15} = \frac{15}{2}\left[2(6) + 14(-4)\right]$

$= -330$

(b) $r = \frac{1}{2} \qquad a = \frac{1}{2}$

$\therefore S\infty = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$

(c) $-1 < 2 - 3x < 1$

$1 > 3x - 2 > -1$

$3 > 3x > 1$

$1 > x > \frac{1}{3}$

QUESTION 7

(a) $0{,}06 = \left(1+\frac{r}{12}\right)^{12} - 1$

$r = 0{,}05841...$

$$F = \frac{2\,500\left[\left(1+\frac{0{,}058}{12}\right)^{120} - 1\right]}{\frac{0{,}058}{12}}$$

$= \text{R}410\,660{,}73$

(b) $$3\,000\,000 = \frac{60\,000\left[1-\left(\frac{0{,}075}{4}\right)^{-n}\right]}{\frac{0{,}075}{4}}$$

$n \approx 149{,}253...$

$\therefore 150$ quarters

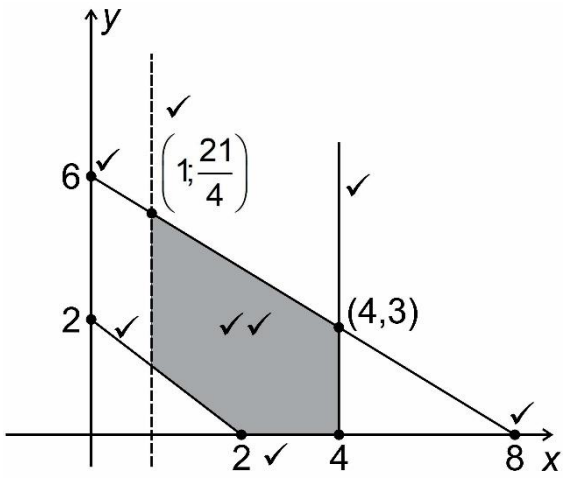
$\frac{150}{4} = 37$ years and 2 quarters

$\therefore$ 1 August 2060

SECTION B

QUESTION 8

(a)



(b) $P = 3(4) - 2(0)$

$= 12$

Writing down gradient of search line earns two marks.
Testing at least three vertices including correct one earns 2 marks

QUESTION 9

(a) $t = 0 : h = 3\text{ m}$

(b) $\frac{dh}{dt} = -2t + 6$

$0 = -2t + 6$

$\therefore t = 3$ at max height

$h_{min} = -3^2 + 6(3) + 3$

$= 12\text{ m}$

QUESTION 10

(a) $f(x) = -2x + 12$

$$g(x) = \frac{a}{x-2} + 3$$

$$0 = \frac{a}{4} + 3$$

$$a = -12$$

(b) $$\frac{-12}{x-2} + 3 = -2x + 12$$

$$\frac{-12}{x-2} = 9 - 2x$$

$$-12 = 9x - 2x^2 - 18 + 4x \quad \text{LCD}$$

$$\therefore 2x^2 - 13x + 6 = 0$$

$$(2x-1)(x-6) = 0$$

$$\therefore x = \frac{1}{2}$$

$$y = -2\left(\frac{1}{2}\right) + 12$$

$$= 11$$

(c) $x \leq \frac{1}{2}$ or $2 < x \leq 6$

(d) $k > -\frac{1}{2}$ or $k < -6$

QUESTION 11

(a) $31 = 2 + a + b + 20$

$\therefore a + b = 9 \ldots (1)$

$$\frac{dy}{dx} = 6x^2 + 2ax + b$$

$$0 = 6 + 2a + b \ldots (2)$$

$\therefore a + (-2a - 6) = 9$

$\therefore -a = 15$

$\therefore a = -15$

$\therefore b = 24$

(b) $0 = 6x^2 - 30x + 24$

$0 = x^2 - 5x + 4$

$0 = (x - 4)(x - 1)$

$\therefore (4;4)$

Point of inflection

$$\frac{d^2y}{dx^2} = 12x - 30 = 0$$

$$x = \frac{5}{2}$$

$$\therefore \left(\frac{5}{2};\frac{35}{2}\right)$$

(c) $x \geq 4$ or $1 \leq x \leq \frac{5}{2}$

QUESTION 12

(a) $a = \frac{\pi r^2}{2} + 2rh$

(b) $4r + 2h = 20$

$\therefore 2r + h = 10$

$\therefore h = 10 - 2r$

$\therefore A = \frac{\pi r^2}{2} + 2r(10 - 2r)$

$A = \frac{\pi r^2}{2} + 20r - 4r^2$

$\frac{dlA}{dr} = \pi r + 20r - 4r^2$

$r(8 - \pi) = 20$

$r = \frac{20}{8 - \pi} = 4{,}12 \text{ cm}$

(c) $\frac{d^2A}{dr^2} = \pi - 8$

$\therefore \frac{d^2A}{dr^2} < 0 \ \therefore$ maximum

QUESTION 13

(a) $\sqrt{2\frac{7}{9}}$

$= \sqrt{\frac{25}{9}}$

$= \frac{5}{3}$

(b) Remainders

1, 0, 1, 0, 1, 0, …, 0 One mark if 0,25 instead of 1

$\therefore 1011 \times 1 = 1011$

(c) $y = x^2$

$\therefore x = \sqrt{y}$

$\therefore y + 4\sqrt{y} - 8 = 0$

$4\sqrt{y} = 8 - y$

$16y = 64 + 16y + y^2$

$\therefore y^2 - 32y + 64 = 0$

(d) $\sum_{x=1}^{100}\left(6x^2 - 2x - 1\right)$

$6\sum_{x=1}^{100} x^2 - 2\sum_{x=1}^{100} x - \sum_{x=1}^{100} 1$

$= \frac{\cancel{6} \times 100(101)(201)}{\cancel{6}} - \frac{\cancel{2} \times 100 \times 101}{\cancel{2}} - 100$

$= 2\,019\,900$

Total: 150 marks