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TOTAL
MARKS

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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2023

MATHEMATICS: PAPER I

EXAMINATION NUMBER

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Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **TWO** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. ONE blank page (page 20) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
21	10	13	10	10	14	9	7	13	12	12	10	9	/150

SECTION A**QUESTION 1**

(a) Solve for x:

(i) $\frac{1}{3}x^2 = -2x$ (3)

(ii) $0,4^{x+2} = 5$ (2)

(b) Show, with reasons, that there are no real values of x for which:

$\log_2(5x+1) - \log_2(x-2) = 2$ (4)

(c) Solve simultaneously for x and y :

$$(x - 3)(y + 1) = 18$$

$$x - 4y = 1$$

(6)

(d) Solve: $f(x) = f'(x)$ where $f(x) = x^2 + 2x$, giving your answer in surd form. (4)

- (e) Write down a possible domain of the function $f(x) = 9 - (x + 4)^2$ such that its inverse will be a function. (2)

QUESTION 2

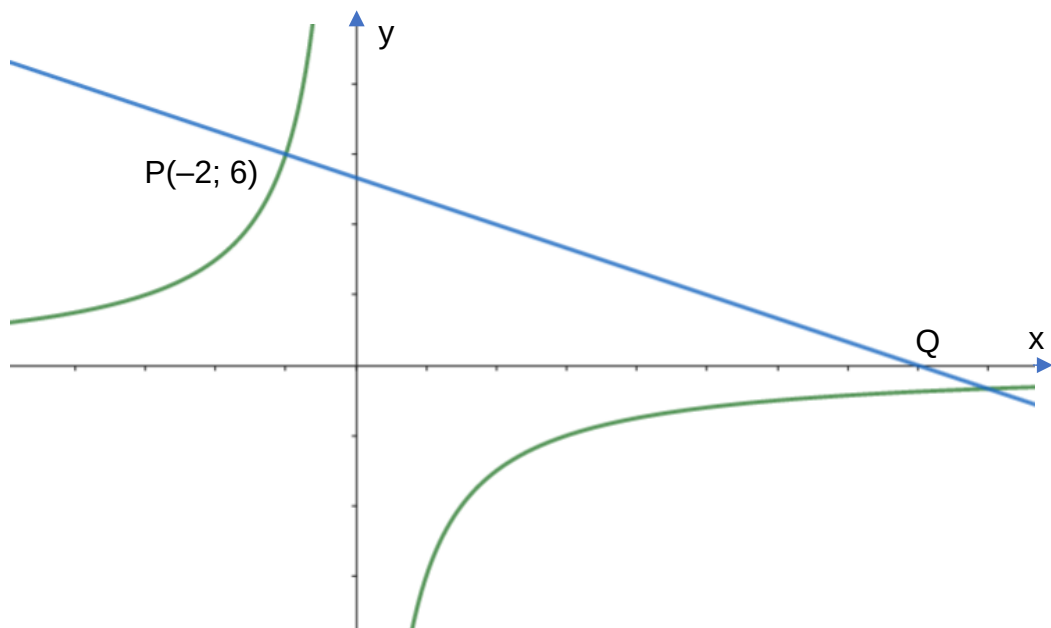
- (a) The first term of a geometric sequence is 10 and the sum to infinity is 30. Determine the common ratio. (3)
- (b) The sum of the 4th and 8th terms of an arithmetic sequence is 36. The sum of the first 20 terms is 675. Find the first term and common difference. (7)

[10]

QUESTION 3

- (a) Find $f'(x)$ from first principles if $f(x) = \frac{3}{2}x^2 - 5$ (5)

- (b) The graph of $f(x) = \frac{a}{x}$ is shown below. Another function, g , a straight line, is perpendicular to the tangent to f at the point $P(-2; 6)$. The graph of g intersects the x -axis at Q .



- (i) Show that $a = -12$. (1)

(ii) Determine the gradient of f at the point P. (3)

(iii) Hence determine the coordinates of Q. (4)

[13]

QUESTION 4

- (a) An investment of x rand increases to R3 600,77 in 5 years, where the interest rate is 7% per annum compounded monthly. Calculate x , to the nearest rand. (4)
- (b) I invest R5 750 per month, starting immediately, over a period of 8 years. The effective rate is 10% per annum.
- (i) Find the nominal interest rate per annum if it is compounded monthly. (2)
- (ii) Calculate the balance in the account after 8 years. (4)

[10]

QUESTION 5

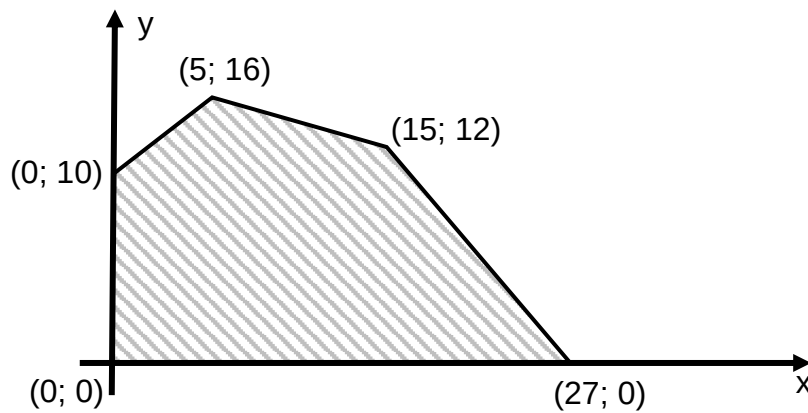
(a) Sketch the graph of $y = 4 - \frac{4}{x-2}$ in the space below. (4)

(b) Sketch the graph of $f(x) = \log_2(x+4)$ and $f^{-1}(x)$ on the same axes in the space below. (6)

[10]

QUESTION 6

A feasible region, R, is defined by the shaded region in the diagram below. The vertices of the region are as indicated in the diagram.



(a) Write down a set of inequalities which describes the region, R. (8)

(b) Maximise P if $P = 3x + 5y$ (3)

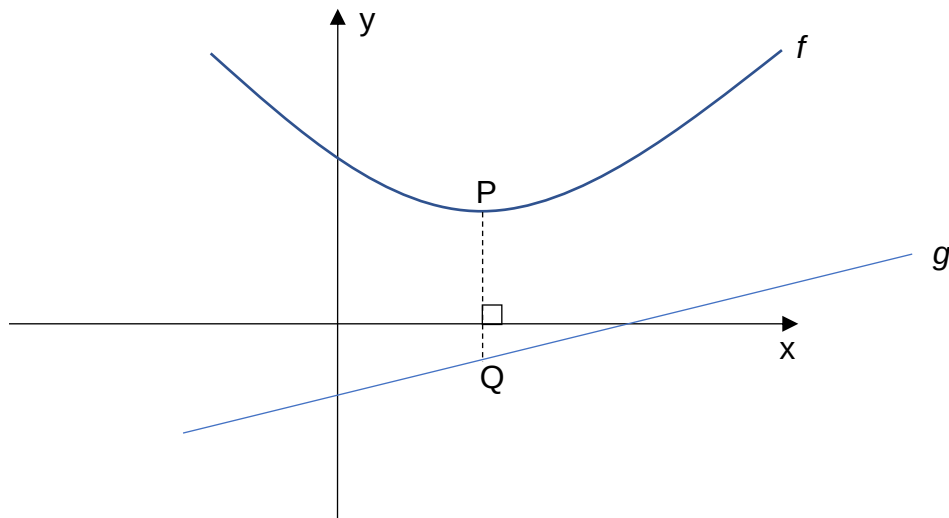
(c) For which values of k will Q be maximised at $(15; 12)$ if $Q = kx + y$? (3)

[14]

78 marks

SECTION B**QUESTION 7**

In the diagram below, $f(x) = x^2 - 6x + 11$ and $g(x) = \frac{1}{2}x - 4$. P lies on f , while Q lies on g . PQ is perpendicular to the x-axis.



(a) Determine the length PQ, if P is the turning point of f . (4)

(b) Find the minimum possible length of PQ. (5)

[9]

QUESTION 8

For which value(s) of k will the equation $\frac{4}{x+k} = -6 - x$ have two real, unequal roots? (7)

[7]

QUESTION 9

- (a) The amount of rainfall, per month (n), in a particular year happens to follow a sequence with formula:

$$T_n = an^2 + bn + c \quad 1 \leq n \leq 12$$

The rainfall for the first three months is given in the table below:

Month	Jan ($n = 1$)	Feb ($n = 2$)	Mar ($n = 3$)
Rainfall (mm)	68	47	32

- (i) Determine the values of a , b and c respectively. (3)

- (ii) Determine the rainfall in December ($n = 12$). (2)

- (iii) In which month was the lowest rainfall? Show clear working. (3)

(b) Solve for p if: $\sum_{k=p}^{4p-1} (2k-1) = 216$ (5)

[13]

QUESTION 10

The function $f(x) = \frac{2}{9}x^3 + ax^2 + bx$ has a stationary point at (6, 12)

(a) Calculate a and b . (7)

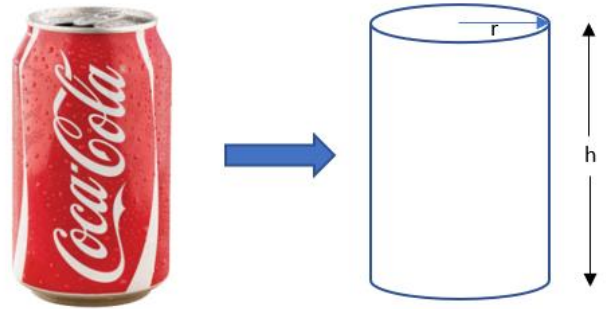
(b) Determine the x-coordinates of the other stationary point and the point of inflection, respectively. (5)

[12]

QUESTION 11

A can of Coke is modelled as the shape of a cylinder, with radius r , height h and a volume of 340 cm^3 (although it is not an exact cylinder in reality).

The cost of making the top and bottom surface of the can is 30 cents/ cm^2 , whilst the curved surface costs 10 cents/ cm^2 .



N.B. Volume of a cylinder = $\pi r^2 h$ Curved surface area = $2\pi r h$

(a) Obtain an expression for h , in terms of r . (2)

(b) Hence show that the outer surface area of the can is given by the formula:

$$A = 2\pi r^2 + \frac{680}{r} \quad (4)$$

- (c) Now write down an expression for the cost of making a can, and hence determine the radius of the can that will lead to a minimum cost. (6)

[12]

QUESTION 12

A loan of R800 000 is taken over a period of 10 years. Interest is charged at 10% per annum, compounded monthly. The first payment is made one month after the granting of the loan.

(a) Calculate the monthly payment. (4)

(b) Determine the outstanding balance after 5 years. (4)

(c) Hence find the interest paid in the 61st month. (2)

[10]

QUESTION 13

- (a) Note that the remainder when 2^4 is divided by 5 is 1, since $\frac{16}{5} = 3 \text{ rem } 1$.

Determine the remainder when 2^{2023} is divided by 5. (5)

- (b) It is given that

$$f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \text{and so on}$$

where $-1 < x < 1$.

Calculate $f'\left(\frac{1}{2}\right)$ (4)

[9]

72 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**