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TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2024

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

EXAMINATION NUMBER

Time: 1 hour   100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 36 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. This question paper consists of THREE modules. Choose **ONE** of the **THREE** modules and tick (✓) the one you have chosen.

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)
3. **Answer the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
5. All necessary calculations must be clearly shown and writing must be legible.
6. Diagrams have not been drawn to scale.
7. **Rounding of final answers.**
   MODULE 2: Four decimal places, unless otherwise stated.
   MODULE 3: Two decimal places, unless otherwise stated.
   MODULE 4: Two decimal places, unless otherwise stated.
8. Four blank pages (pages 33–36) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR MARKER'S USE ONLY

Module 2	Q1	Q2	Q3	Q4	Q5	**Total**
Marks	**16**	**16**	**31**	**23**	**14**	**100**

Module 3	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	**20**	**17**	**13**	**14**	**25**	**11**	**100**

Module 4	Q1	Q2	Q3	Q4	Q5	**Total**
Marks	**20**	**29**	**14**	**21**	**16**	**100**

MODULE 2 STATISTICS

QUESTION 1

1.1 Moi collects a basket of plums, however only 80% of them are ripe. The probability that a ripe plum has a worm inside is p. The probability that an unripe plum has a worm inside is 0,1.

(a) Complete a tree diagram in terms of p.

(4)

(b) Find the value of p, if exactly half of all the plums have worms inside.

(3)

1.2 The random variable $X \sim B(n\,;\,p)$ can be approximated by $Y \sim N(\mu\,;\,\sigma^2)$ when certain conditions are fulfilled.

(a) Explain why a continuity correction must be applied when using the normal approximation to a binomial distribution.

(2)

(b) For each of the following binomial random variables, X:

- State, with reasons, whether X can be approximated by a normal distribution.
- If so, write down the normal approximation to X in the form $N(\mu\,;\,\sigma^2)$.

(1) $X \sim B(70\ ;\ 0{,}04)$

(2) $X \sim B(70\ ;\ 0{,}4)$

(7)
[16]

QUESTION 2

2.1 Mr Crosse has a keen interest in garden birds, in particular, the Bronze Mannikin. The mean length of these birds is μ cm. He takes a random sample of 40 of these birds and finds that a 90% confidence interval for μ is 8,3 cm to 9,9 cm.

(a) Calculate the sample mean and the standard deviation, correct to 2 decimal places.

(6)

(b) Mr Crosse takes 100 samples and calculates a 95% confidence interval for each one. Approximately how many of these confidence intervals will not contain the population mean?

(2)

2.2 A pharmaceutical company claims that a certain headache pill contained less than 300 mg of aspirin. Tests carried out on 120 tablets resulted in a sample mean of 298 mg, with a standard deviation of σ mg.

(a) State the null and alternate hypothesis.

(2)

(b) If the pharmaceutical company's claim was rejected at the 1% level of significance, find the largest possible value of σ, correct to 1 decimal place.

(6)

[16]

QUESTION 3

A particular model of digital camera has a lifespan that is normally distributed with a mean of 6,5 years and a standard deviation of 1,15 years.

(a) A customer buys this model camera. Show, with all calculations, that the probability that the camera lasts longer than 5 years is approximately 90%.

(6)

(b) 5% of these cameras last less than t years. Calculate the value of t correct to the nearest month.

(6)

The manufacturer offers a five-year warranty for this model of camera. The probability of a claim against the warranty remains at 10%.

Ten digital cameras of this model are sold on a certain day.

(c) Find the probability that at most one of them will claim against the warranty.

(8)

(d) Find the probability that the tenth camera sold will be the second one to be claimed against the warranty.

(5)

The company's policy is such that cameras that develop a fault in less than 3 years will be replaced with a new camera.

(e) Given that a camera is claimed for warranty, find the probability that it has been replaced with a new one.

(6)
[31]

QUESTION 4

4.1 The discrete random variable X has the probability distribution shown in the following table:

x	0	1	2	3	4
$P(X=x)$	p	p	0,2	0,1	q

It is given that $E[X]=2{,}45$.

(a) Find the values of p and q.

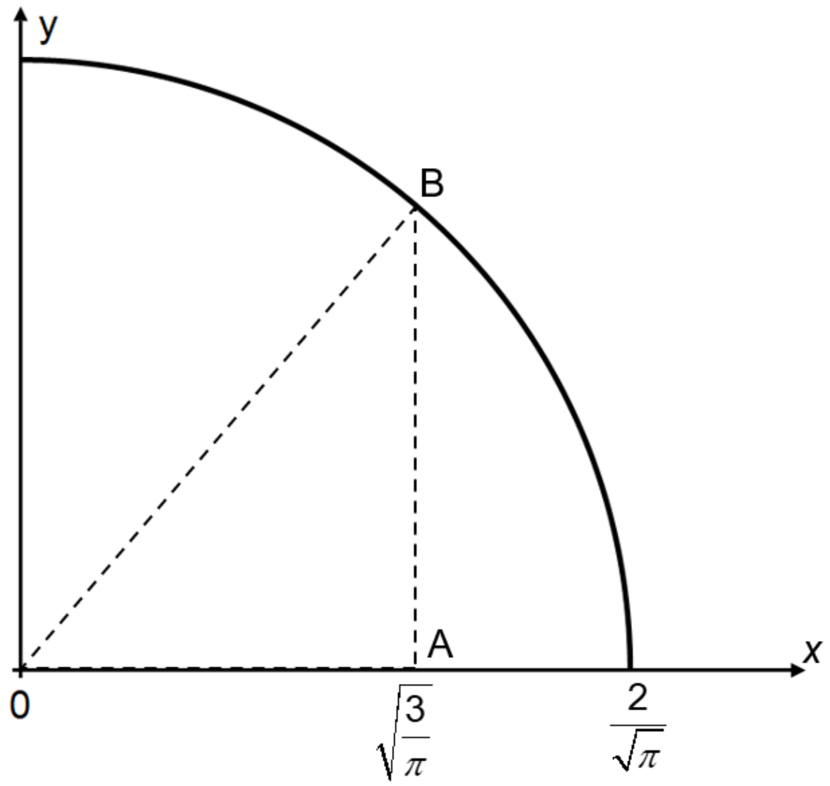
(6)

(b) Find $P(X<E[X])$.

(3)

4.2 A random variable X has probability density function f,

- where the graph of $y = f(x)$ is a quarter circle with centre (0;0) and radius $\dfrac{2}{\sqrt{\pi}}$, $x \geq 0\ ;\ y \geq 0$.
- Elsewhere $f(x) = 0$.



(a) Verify that $f(x)$ is a probability density function.

(4)

A and B are the points where the line $x = \sqrt{\dfrac{3}{\pi}}$ meets the x-axis and the quarter circle respectively.

(b) Show that $A\hat{O}B = \dfrac{\pi}{6}$ *radians*.

(4)

(c) Hence, find $\text{P}\left(X < \sqrt{\dfrac{3}{\pi}} \right)$.

(6)

[23]

QUESTION 5

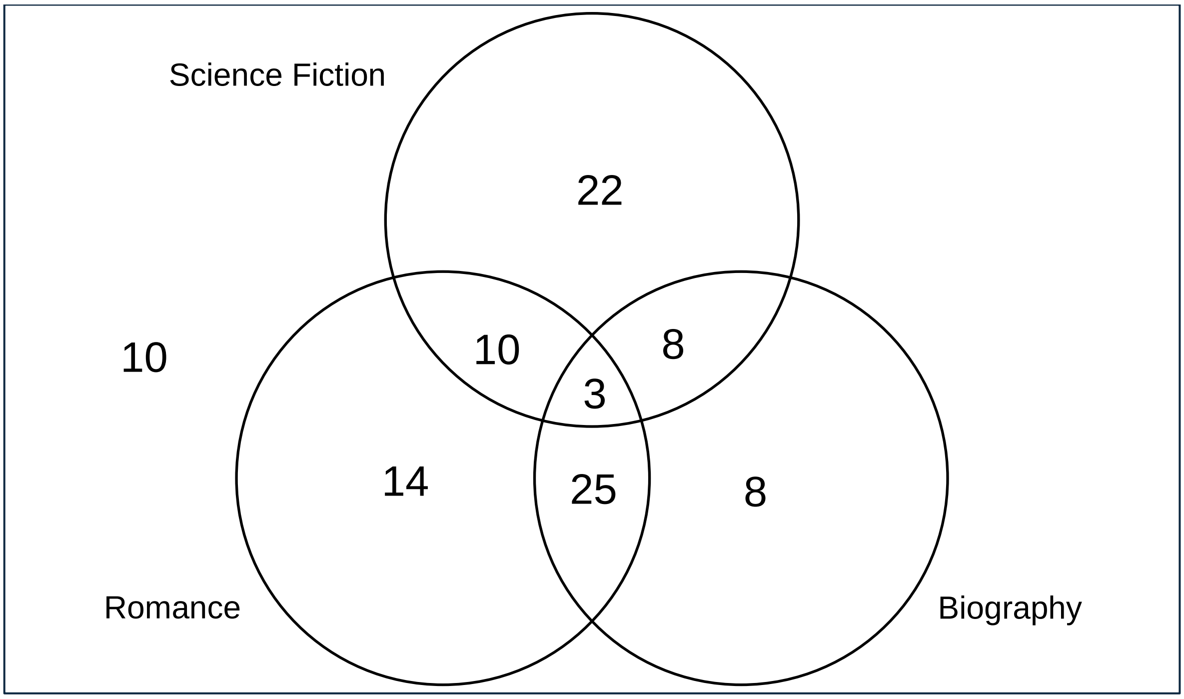
5.1 A group of 12 people consist of:

- 3 boys
- 4 girls
- 5 adults

Determine the probability that a randomly selected team of 5 people from the group will have exactly one adult and at least 1 girl and 1 boy.

(7)

5.2 The Venn diagram shows the numbers of students reading various types of books in a grade group of 100 students.



A student is chosen at random from the 100 students. Then another student is chosen from the remaining students.

Find the probability that the first student reads Biography books and the second student reads Romance but not Science Fiction.

(7)
[14]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Shailen takes out a loan of R15 000 now, and a further Rx, 6 years from now. The loan is to be repaid by means of semi-annual instalments of Ry, starting in 6 months' time and finishing in 10 years' time.

(a) Calculate x if $y = 3\ 500$ and the interest rate is constant at 10% per annum compounded semi-annually.

(8)

(b) Calculate y, given that x = R10 000 and the interest is 10% per annum compounded semi-annually for the first 6 years and 12% per annum compounded semi-annually for the remaining 4 years.

(12)
[20]

QUESTION 2

A loan is taken and is to be repaid by means of 120 monthly payments, starting after one month, with interest calculated at 12% per annum, compounded monthly. The total interest paid over the course of the loan is R1 443 302,76.

(a) Calculate the monthly payment, and show that the loan amount is R2 000 000.

(10)

(b) Calculate the interest paid in the 81^{st} month.

(7)

[17]

QUESTION 3

A savings scheme, where interest is calculated monthly, is represented by the following recursive equation:

$$F_{n+1} = aF_n + 1200\,(1{,}08)^n\,; \quad F_0 = b$$

(a) After 1 month the balance is R3 230 and after 2 months it is R4 574,45.
Calculate *a* and *b*.

(9)

(b) Determine the effective per annum interest rate.

(4)
[13]

QUESTION 4

A population, P_n, is growing logistically within an environment with a fixed carrying capacity, K. At the point at which the population is growing at its most rapid rate, P_n = 6 000. Furthermore, when P = 10 000 then $\frac{\Delta P}{P} = 0{,}03$.

(a) Sketch the graph of $\frac{\Delta P}{P}$ against P, indicating at least two points on the graph.

(3)

(b) Prove that: $P_{n+1} = 1{,}18P_n - 0{,}000015P_n^2$

(9)

(c) Determine $\dfrac{\Delta P}{P}$ when the growth rate is at its greatest.

(2)
[14]

QUESTION 5

There is a fascinating relationship between the great white shark and the Cape fur seal in the waters around Seal Island, off the coast of Cape Town. The sharks employ a hunting technique known as 'breaching', where they launch themselves vertically out of the water to catch seals swimming near the surface.

Great white sharks, of which 30% are thought to be reproducing, typically have a small litter every two years. The great whites have a long lifespan, but this can be taken to be 12 years in terms of their interaction with the island. There are currently 6 regular predators around the island. Each shark may eat up to 45 seal pups per year.

Female Cape fur seals typically produce a single pup every nine months, with a 15% pup mortality rate. Assume that one-quarter of the seal population is producing young and there are currently around 1 500 in the waters around the island, with a capacity for 1 650. With a prevalence of sharks, the equilibrium of seals is predicted to be 228. Consider a yearly cycle.

(a) Calculate the intrinsic growth rate of the seals, to 4 decimal places.

(6)

(b) Determine the rate of successful attacks of the sharks on the seals.

(5)

(c) Calculate an estimate of the litter size of the great white shark.

(8)

(d) Determine the equilibrium number of sharks.

(6)
[25]

QUESTION 6

Two simultaneous recursive equations are given as follows:

$$y_{n+1} = \frac{3y_n}{5\,000}\left(1250 + x_n\right) \qquad y_0 = 100$$

$$x_{n+1} = \frac{x_n}{1000}\left(1500 - x_n - 20y_n\right) \qquad x_0 = 10$$

(a) Calculate $(x_1; y_1)$

(3)

(b) Determine the equilibrium values of x and y.

(8)
[11]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 State the nature of the transformation or transformations produced when multiplying by the following matrices.

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(2)

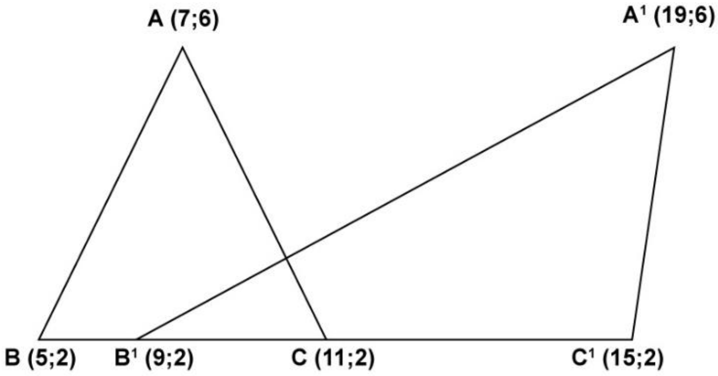
(b) $\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

(3)

(c) $\begin{bmatrix} 0 & -3 \\ -1 & 0 \end{bmatrix}$

(4)

1.2 Given:



(a) Find the transformation matrix that maps ABC onto $A^1B^1C^1$.

(7)

(b) Name the transformation.

(2)

(c) Determine the ratio:

$$\frac{Area(ABC)}{Area(A^1B^1C^1)}$$

(2)

[20]

QUESTION 2

2.1 (a) A matrix must be a __________________ matrix, for the determinant to exist. (1)

(b) The inverse of a matrix only exists if the determinant is

__________________________. (2)

2.2 Determine the value of x given

$M = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$; $P = \begin{pmatrix} 2 & x \\ 3 & 1 \end{pmatrix}$; $Q = \begin{pmatrix} 3x+2 & 7 \\ 7-x & 7 \end{pmatrix}$

and $M^T \times P^T = Q$.

(6)

2.3 A set of equations is given:

$2x - 5y + 3z = 8$

$3x - y + 4z = 7$

$x + 3y + 2z = -3$

The row reduction of the associated augmented matrix has been started below.

$$\left[\begin{array}{ccc|c} 2 & -5 & 3 & 8 \\ 3 & -1 & 4 & 7 \\ 1 & 3 & 2 & -3 \end{array}\right] \; 2R_2 - 3R_1$$

$$\sim \left[\begin{array}{ccc|c} 2 & -5 & 3 & 8 \\ 0 & 13 & -1 & -10 \\ 1 & 3 & 2 & -3 \end{array}\right]$$

Continue the row reduction, to solve for *x*, *y* and *z*.

(7)

2.4 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$, find:

(a) the determinant of A.

(4)

(b) the matrix of cofactors of A.

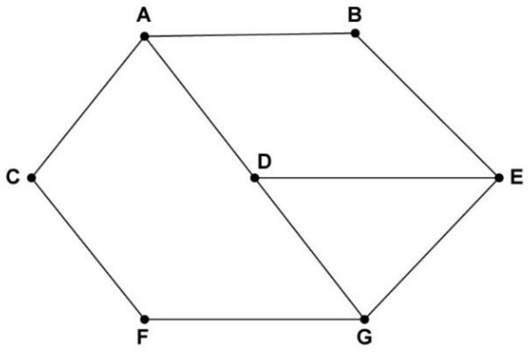
(5)

(c) the inverse of A.

(4)
[29]

QUESTION 3

3.1 Explain clearly what you would have to do to create an Eulerian circuit in the graph below.



(3)

3.2 Given P =

	A	B	C	D	E
A	0	1	1	0	1
B	1	0	0	0	0
C	1	0	0	1	1
D	0	0	1	0	1
E	1	0	1	1	0

Draw the graph associated with matrix P.

(5)

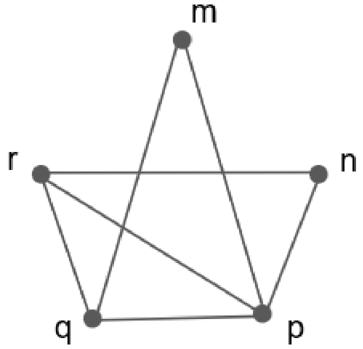
3.3 Given two simple connected graphs.

Graph (i)

Set of vertices = {a; b; c; d; e}

Set of edges = {(a, b); (a, c); (a, e); (b, c); (b, d); (b, e); (c, d); (d, e)}

Graph (ii)



(a) Give a reason why Graph (i) is not isomorphic to Graph (ii).

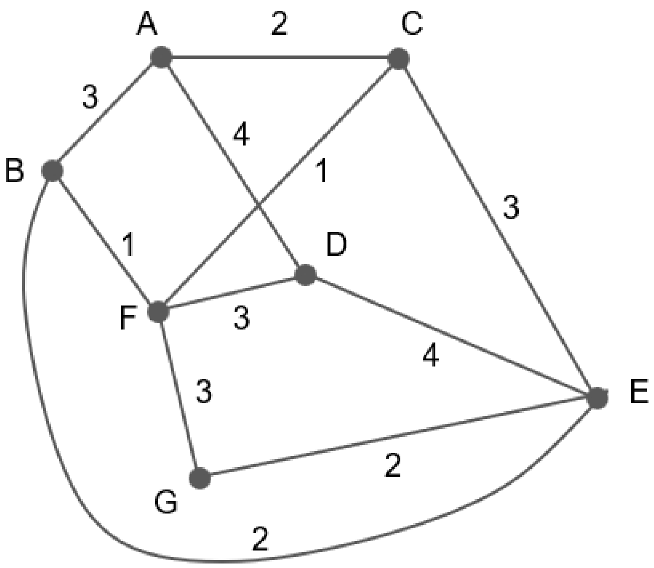
(2)

(b) Name one edge that could be removed from one of the graphs to make the two graphs isomorphic.

(4)
[14]

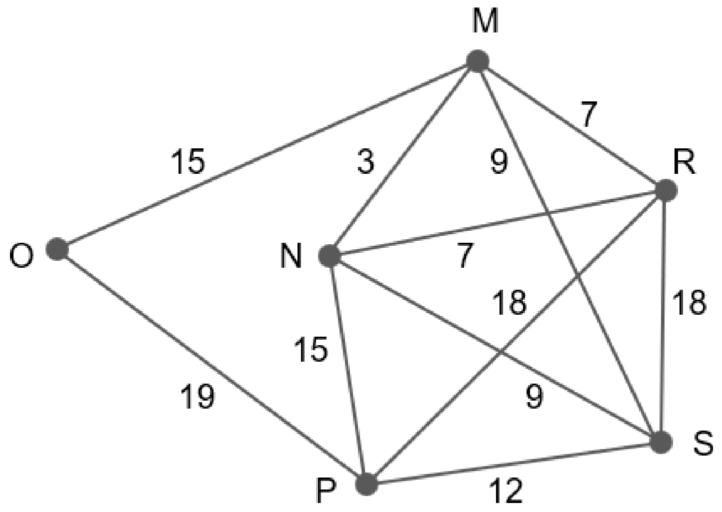
QUESTION 4

4.1 Use Prim's algorithm to find the minimum spanning tree for the graph, starting at vertex A. Clearly state the order in which you choose the edges, as well as the total length of the spanning tree.



(8)

4.2 (a) A delivery route, starting and ending at O, must visit the five locations labelled M, N, P, R and S. Use the nearest neighbour algorithm to find the upper bound for the delivery route. Clearly show the order of the visited vertices.



(8)

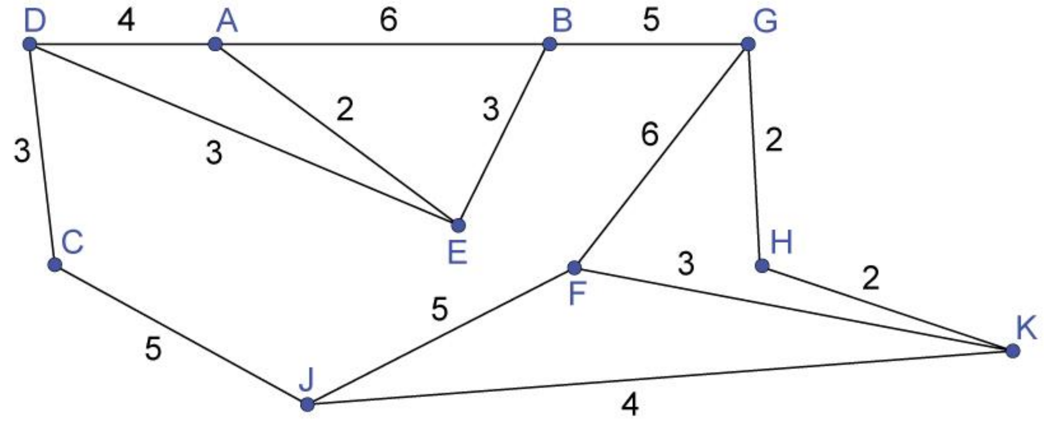
(b) Find one delivery route, starting and ending at O, that is shorter than the upper bound.

(5)

[21]

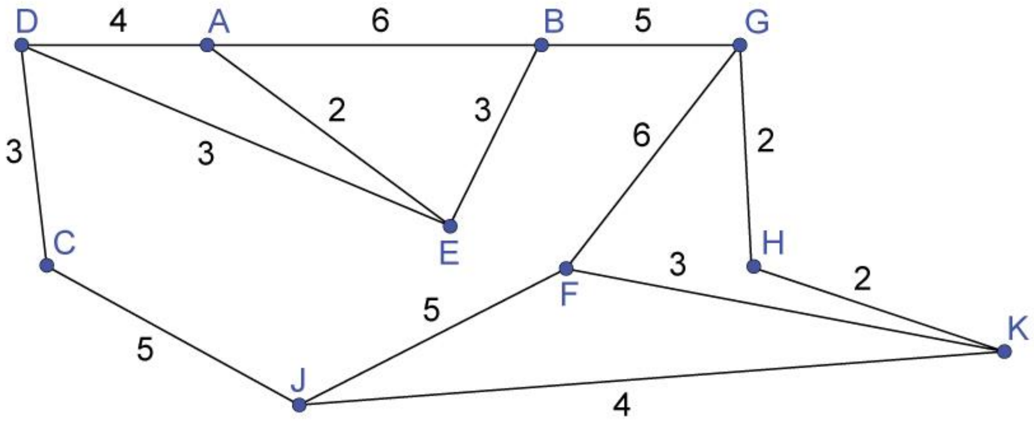
QUESTION 5

A house is renovated into a hotel. The edges represent the distance between rooms in metres. The rooms are represented by the vertices.



(a) Use Dijkstra's algorithm to find the shortest path from A to K. Show clear evidence of your working, including the termination of non-viable routes. Be sure to state your final route, as well as its length.

(8)



(b) A locked door blocks CJ.
With the aid of your answer in Question 5 (a), find the shortest route from A to J.

(3)

(c) An alteration to the hotel connects B to F, such that the triangle inequality holds. The alteration means that AEBFK is now the shortest route from A to K.

Determine the possible length(s) of BF.

(5)
[16]

Total for Module 4: 100 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.