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TOTAL
MARKS

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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2024

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

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Time: 2 hours

200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.
7. Two blank pages (pages 23 to 24) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Mark												
Marker Initial												
Moderated Mark												
Moderator Initial												
Question Total	43	12	10	27	16	12	10	16	26	14	14	/200

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$:

(a) $x \leq \frac{4}{x}$

(8)

(b) $\ln(fe^x + c) = k$

(6)

1.2 Solve for $x \in \mathbb{R}$.

$$|x^2 + 6x| = 8$$

(6)

1.3 One of the factors of $f(x) = 2x^5 + 5x^4 + 13x^3 + 15x^2 - 2x - 3$ is $(2x + 3)$.
Find the corresponding factor.

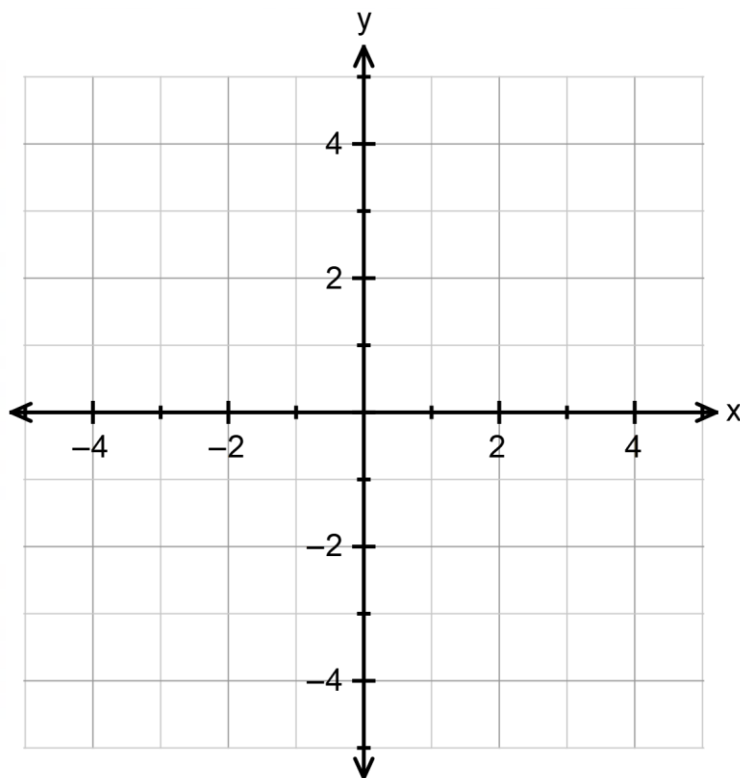
(7)

1.4 Determine real values of a and b if:

$$(a + bi)(4 - i) = a + 3bi + 3b + i$$

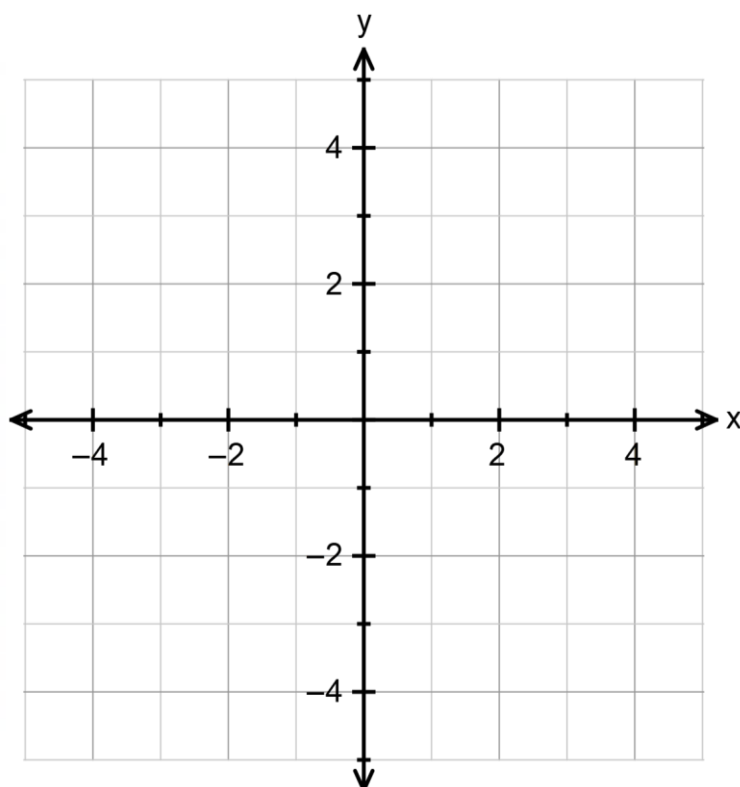
- 1.5 Sketch the following functions on the axes provided. Also indicate the equations of any asymptotes and show any intercepts with the axes.

(a) $y = |\ln(x+1)|$



(4)

(b) $y = \ln|x-1|$



(4)
[43]

QUESTION 2

Use mathematical induction to prove that:

$6^n - 1$ is divisible by 5 for all $n \in \mathbb{N}$

QUESTION 3

If $f(x) = \frac{1}{\sqrt{x}}$ then determine $f'(x)$ by first principles.

[10]

QUESTION 4

- 4.1 Find, with classification, all asymptotes of the following rational functions. You should give the value(s) of x at which any removable discontinuities exist.

(a) $y = \frac{2x^2 + 6}{x^2 - 4}$

(5)

(b) $y = \frac{x^2 + 4x + 3}{2x + 1}$

(6)

(c) $y = \frac{x^2 - 6x + 8}{x^3 - 2x^2 - 8x}$

(6)

4.2 Consider the function: $f(x) = \frac{x^2 + 2x - 5}{2x - 3}$.

(a) Determine the coordinates of its stationary points.

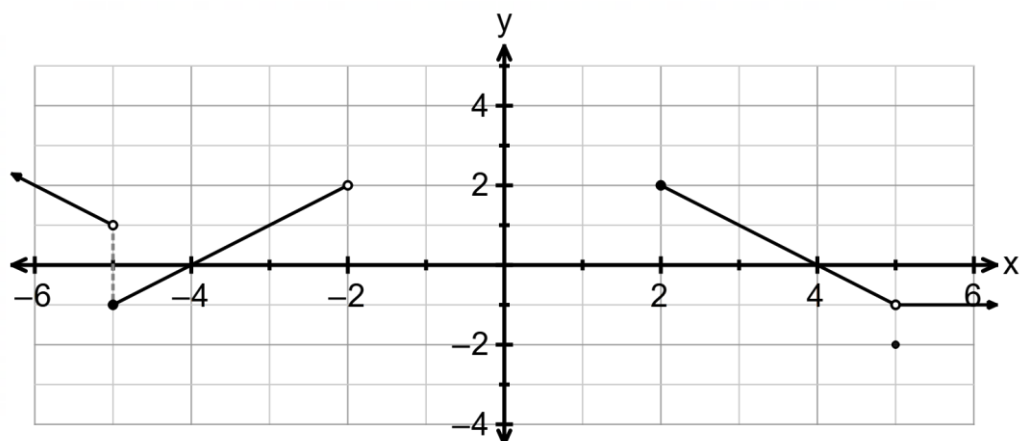
(8)

(b) Use the fact that $f''(x) = \frac{4x - 6}{(2x - 3)^4}$ to justify the nature of the stationary points you found in Question 4.2 (a).

(2)
[27]

QUESTION 5

Consider the function f shown below:



(a) Write a mathematical statement to justify why f is discontinuous at ...

(1) $x = -5$

(2)

(2) $x = 5$

(2)

(b) Classify the discontinuity that exists at:

(1) $x = -5$

(2)

(2) $x = 5$

(2)

(c) Suppose $g(x) = \begin{cases} f(x) & -4 \leq x < -2 \\ ax^2 + bx + c & -2 \leq x < 2 \\ f(x) & 2 \leq x \leq 4 \end{cases}$

Find a , b and c if $g(x)$ is differentiable at every point on the interval $[-4 ; 4]$.

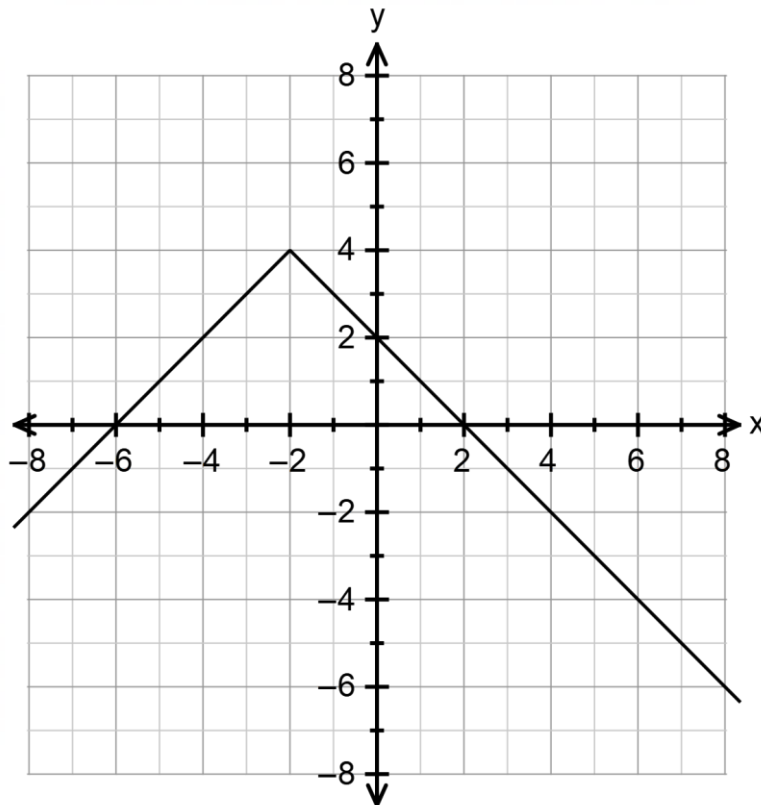
QUESTION 6

6.1 The function $y = -|x + 2| + 4$ has been drawn below.

Draw another function which will enable you to solve:

$$|x + 2| \geq -|x + 3| + 7 \text{ graphically.}$$

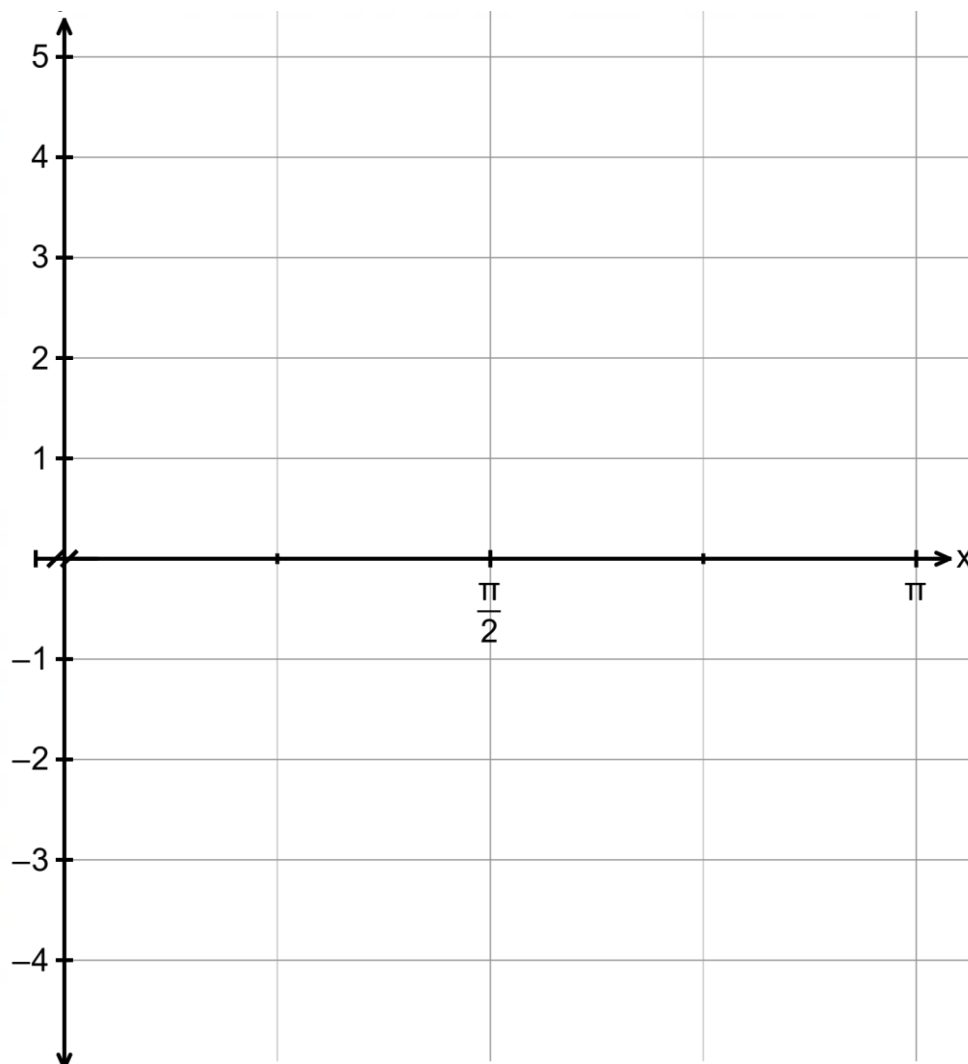
Hence, or otherwise, give the solution to the inequality.



(8)

6.2 Draw the graph of $y = |\tan x|$ on the axes provided showing all points of interest.

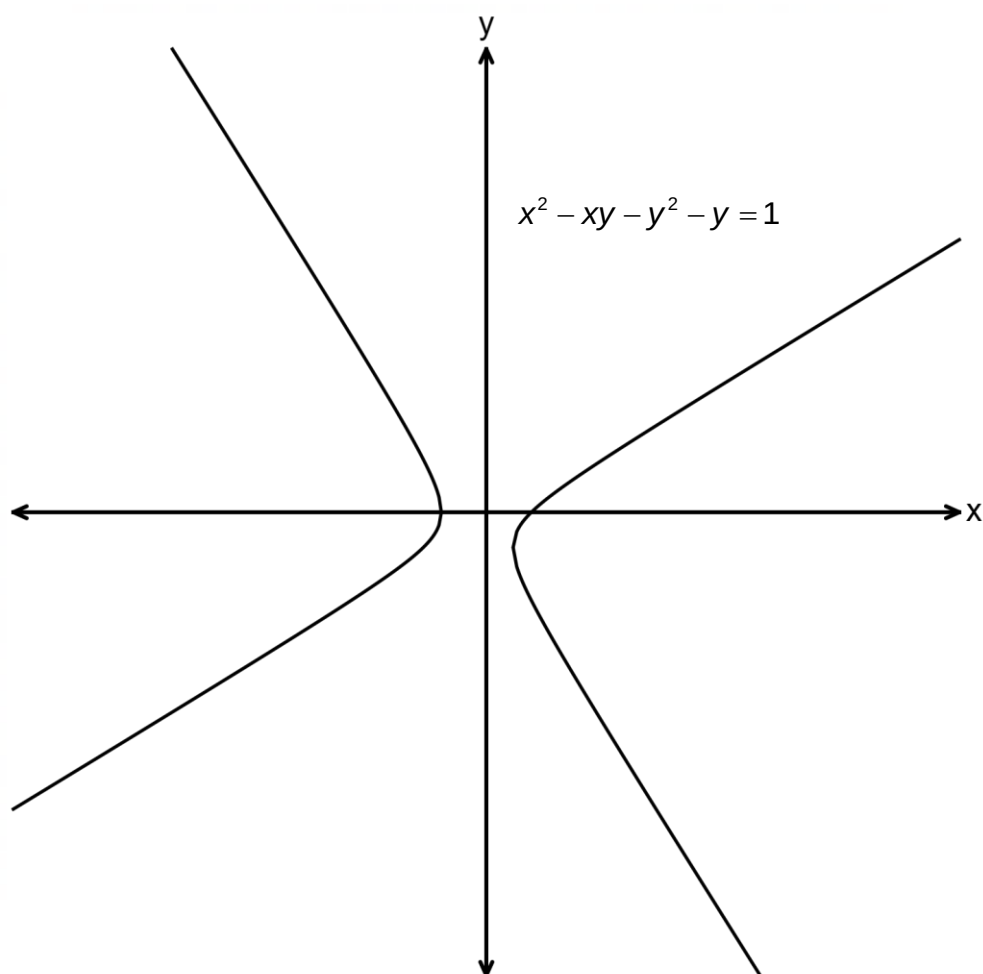
You should also draw and label any asymptotes with their equations.



(4)
[12]

QUESTION 7

A portion of the graph of the implicitly defined relationship $x^2 - xy - y^2 - y = 1$ is shown below.



- (a) Determine the coordinates of the positive x-intercept.

(2)

- (b) Find the equation of the tangent to the curve at the point found in Question 7 (a).

QUESTION 8

- (a) You have learnt how to differentiate e^x and $\ln x$ but what about differentiating exponential and log functions of other bases?

$$\text{Given } a^x = e^{\ln a^x} = e^{x \ln a} \text{ and } \log_a x = \frac{\ln x}{\ln a}$$

Hence, determine:

$$\frac{d}{dx}(2^x + \log_3 x)$$

(8)

- (b) Solve $\cos^2\left(\theta - \frac{\pi}{6}\right) - \sin^2\left(\theta - \frac{\pi}{6}\right) = -\frac{1}{2}$ for $\theta \in [0; 2\pi]$

(8)
[16]

QUESTION 9

Determine the following integrals:

(a) $\int x \sec^2(5x^2) dx$

(8)

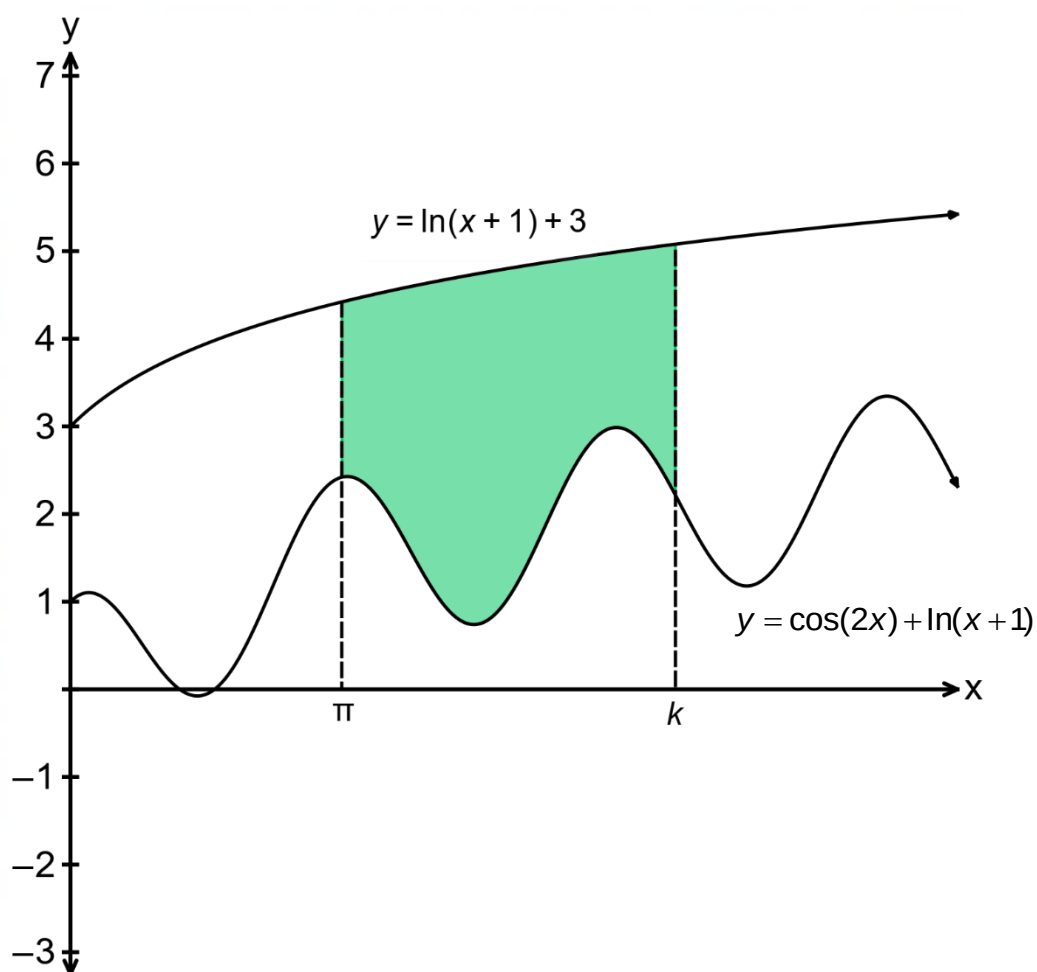
(b) $\int 3 + \cot^2 x dx$

(8)

(c) $\int \frac{x^3 + 4}{x^2 - 4} dx$

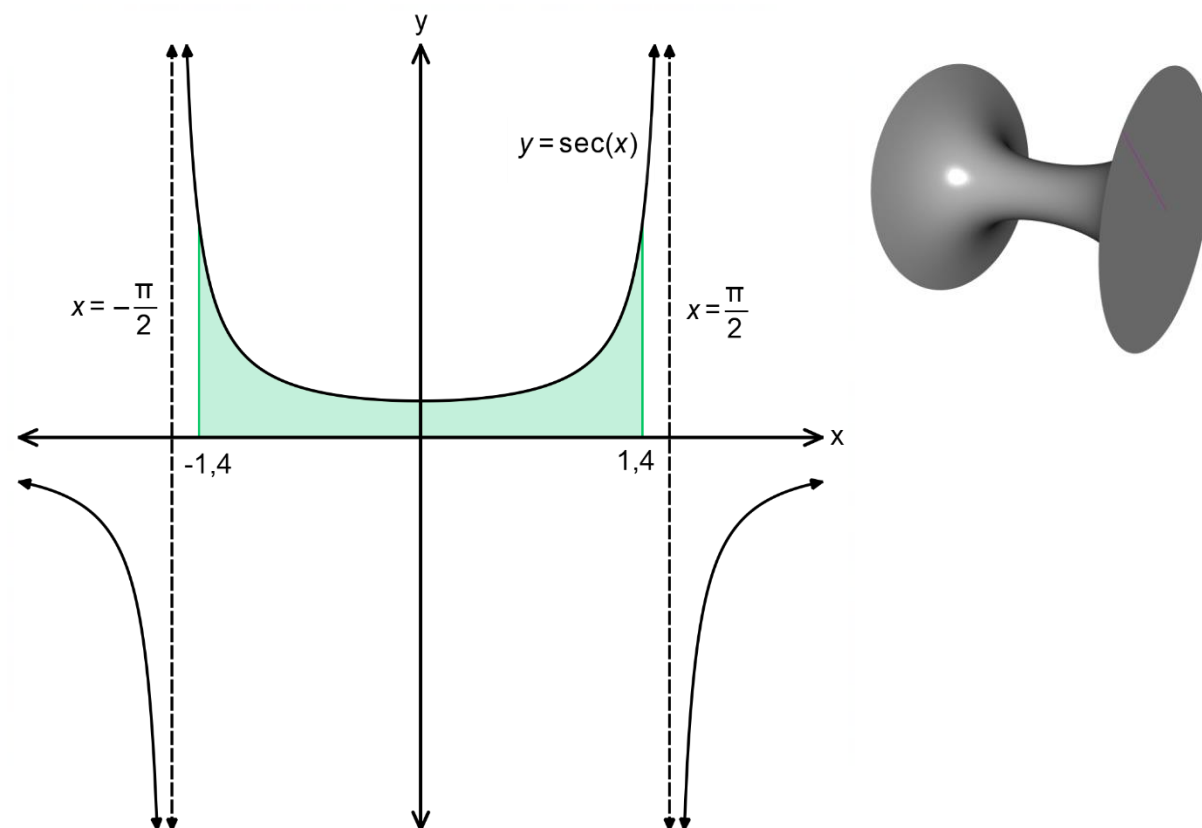
QUESTION 10

- (a) Determine the shaded area in terms of k showing all working details. You should evaluate any integrals in your answer.



(8)

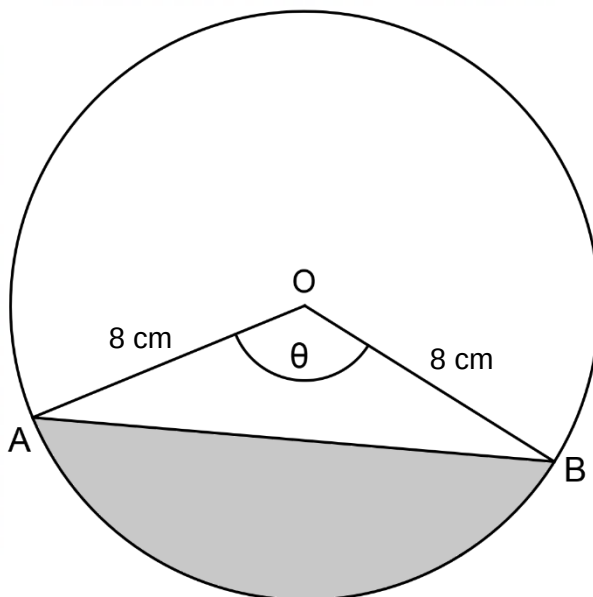
- (b) Determine, to 1 decimal place, the volume when the shaded area below is rotated about the x -axis.



(6)
[14]

QUESTION 11

In the diagram below, O is the centre of the circle. $\hat{AOB} = \theta$ and $AO = BO = 8$ cm. The shaded area is 45 cm^2 .



Use the Newton-Raphson method to determine θ (in radians) to 5 decimal places.

You should:

- Show the equation you are solving.
- Show the iterative formula you will use.
- Use an initial approximation, $\theta_0 = 1$.
- Give θ_1 to 5 decimal places.

[14]

Total: 200 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

